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A MOMENT PLOTTING METHOD FOR CHARACTERIZING AIRCRAFT FATIGUE DATA POPULATION DISTRIBUTIONS

by

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of the requirements for the degree of

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A method was developed where, by statistically analyzing samples of aircraft fatigue data from a population, the population's probability distribution function is determined. The method uses plots of a sample's coefficient of variation, and measures of skewness and kurtosis, superimposed on templates of curves created from the moment functions of Normal, Lognormal, Weibull and Exponential distributions. The moment functions are derived for each distribution. The plotting technique is combined with a comparison of goodness of fit statistics to form the complete distribution selection method. The method is first tested on random numbers generated from known distributions to see if the correct distributions are selected; then the method is applied to helicopter loads data, fatigue life characterization test data, and counting accelerometer data.

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I. INTRODUCTION

A. PURPOSE AND OBJECTIVE

The purpose of this thesis was to investigate a method of moment comparisons to determine statistical descriptions of aircraft structural fatigue characteristics through appropriate selection of probability distribution functions. Once the population distribution and it's parameters are determined, they can be used to model fatigue parameters such as applied loads that are damaging, static and fatigue lives of material characterization samples, full scale fatigue tests, air gust load intensity, sink speeds for carrier landings, etc.

Realities are such that in many of these instances the sample size may be rather small, statistically speaking. The full scale fatigue test is a sample size of one because of cost, and yet some estimates about other aircraft in the population must be made from it. This represents the extreme, but there are many instances in helicopter flight substantiation tests where the number of load variations in a maneuver will be on the order of 20. For this reason, a second objective of this thesis was be to study sample size influences on the methods of characterization.

B. BACKGROUND

Fatigue lives are based on two factors: stresses endured by the aircraft, and the material fatigue properties. Both of these factors must be studied through statistical analysis due to random variations in their values.

1. Statistical Nature of Flight Loads

In aircraft fatigue work, fatigue damaging stresses result from loads encountered by the aircraft being studied. In the case of helicopters, these loads are measured during the final design phase through substantiation flights. Critical components and structural members are fitted with strain gages, and a time-load history is recorded. Typically 40 to 60 parameters are recorded in each of a variety of flight regimes which reflect the expected or potential usage and flight envelope of the aircraft. An effort is made to have the same gross weight, center of gravity, airspeed, rotor speed, load factor, density altitude etc., for each flight to minimize variability in the loads produced. However, items like pilot technique, gusty air conditions, and instrument accuracy introduce considerable variations. Because of the high cost of substantiation flights, it is not feasible to gather large amounts of data by flying hundreds of flights to statistically establish the "true" loads encountered in each flight regime. One method used to ensure conservatism is to select the highest load encountered in a given regime, and assume it occurs 100% of the time the aircraft flies that regime. Another method used is to select the 95th percentile load of those recorded in a given regime and assume it occurs 100% of the time the aircraft flies that regime. Both of these methods are likely to be conservative, but they are also subject to "high envelope growth". This occurs when subsequent flights produce higher loads than the established highest load (or a higher 95th percentile load). [Ref. 1] When this occurs it reduces the validity of the fatigue life predictions which were based on earlier substantiation flights. It would be useful to determine the population distribution of the load samples from each flight regime. Statistical models could then be constructed to model the loads for situations not covered by the substantiation flights, or for situations that were not represented accurately during substantiation due to the uncontrollable factors mentioned earlier.

Every Navy aircraft is monitored fatigue-wise. Many are monitored by making load measurements using a variety of instruments. Measured data is recorded in the fleet and sent at regular intervals to designated ground facilities for processing. As a result of missing data, the gaps are filled via statistical representations of loads data from all the other aircraft of that type. This statistical application to the fatigue calculation is dependent upon the size of the fleet of that particular aircraft, which is sometimes small.

2. Statistical Nature of Material Behavior

Even if the stresses or loads to be encountered were known exactly, fatigue life still could not be calculated exactly due to the inherent variations in the strength of the structures or components themselves. Manufacturing processes like stamping, rolling, grinding, machining or heat-treating can cause surface irregularities or residual stresses. Assembly processes such as rivet placement and bolt torque can produce variations in stress concentrations. Even nondominant, microscopic, interstitial impurities or dislocations have a strong effect on fatigue life. Depending on the loads involved, these small irregularities can cause a large scatter in fatigue lives for material samples, subassemblies, and full scale tests, even when subject to identical loading. "Coupon" testing is relatively inexpensive, so sizable amounts of data can be generated, which can be statistically analyzed to characterize the variability. If the data can be fit to a distribution and modeled, various "safe-lives" can be developed to ensure an item will not fail before a certain period of time. These lives are assigned confidence levels, which indicate that a specified percentage of the items would statistically survive the calculated safe-life a specified percentage of the time. [Ref. 2]

Having motivated the broad need of statistical characterization in fatigue analysis of aircraft, the moment method fundamentals will now be developed.

II. DISTRIBUTION MOMENTS METHOD

A. MOMENTS DEFINED

The method of distribution identification developed in this chapter is based on a group of statistics known as moments of the probability distribution function. The first moment of a distribution is more commonly referred to as the ***mean*** μ_x, or the expected value of the random variable X. The expected value is the centroid of the distribution.

$$\mu_x = E[X] = \int_{-\infty}^{+\infty} x f(x) dx \qquad 2.1$$

$E[X]$ is the expected value, or expectance. For a continuous distribution, $f(x)$ is the distribution's probability density function, and x is the value of the random variable. The mean is the first moment about zero. Higher order moments can be calculated about any point in the distribution, but are most commonly taken about zero (the raw moment), or about the mean (the central moment). The second central moment $\mu_x^{(2)}$ is known as the variance, and is a measure of the distribution's dispersion. It is akin to the moment of inertia in dynamics or structures. The square-root of the variance is the more commonly used standard deviation, denoted by the symbol σ. Equation 2.2 is the expression for the second moment about the mean.

$$\mu_x^{(2)} = E\left[(X-\mu_x)^2\right] = \int_{-\infty}^{+\infty} (x-\mu_x)^2 f(x)dx \qquad 2.2$$

Equation 2.3 is the general form for the rth order, central moment and is defined as expected:

$$\mu_x^{(r)} = E\left[(X-\mu_x)^r\right] = \int_{-\infty}^{+\infty} (x-\mu_x)^r f(x)dx \qquad 2.3$$

It is important to note that the superscript on μ_x in parentheses is not an exponent.

The expectance term for the variance is expanded as shown below:

$$\mu_x^{(2)} = E\left[(X-\mu_x)^2\right] = E\left[X^2\right] - 2\mu_x E[X] + (\mu_x)^2 \qquad 2.4$$

where the expected value of a constant is that constant, $E[\mu_x] = \mu_x$ and $E[2] = 2$.

The third central moment is called the *skewness* and according to Equation 2.3, it specializes to Equation 2.5. Equation 2.6 is the algebraic expansion of the third moment.

$$\mu_x^{(3)} = E\left[(X-\mu_x)^3\right] \qquad 2.5$$

$$\mu_x^{(3)} = E\left[X^3\right] - 3\mu_x E\left[X^2\right] + 3\mu_x^2 E[X] - \mu_x^3 \qquad 2.6$$

This will be used in derivations to follow. If a distribution has a non zero third moment it is not symmetric about it's mean. If it is negative it possesses a left "tail" and is said to be skewed to the left, or negatively skewed. Similarly, if the moment is positive, the tail is to the right and the distribution is positively skewed.

The fourth, and final, moment used in this thesis is called the *kurtosis*. This is a measure of the peakedness of the distribution. If the tails are long, the kurtosis is greater than if the tails are short. The expression for kurtosis is given in Equation 2.7, the algebraic expansion is provided in Equation 2.8. [Ref. 2 and Ref. 3]

$$\mu_x^{(4)} = E\left[\left(X - \mu_x\right)^4\right] \qquad 2.7$$

$$\mu_x^{(4)} = E\left[X^4\right] - 4\mu_x E\left[X^3\right] + 6\mu_x^2 E\left[X^2\right] - 4\mu_x^3 E\left[X\right] + \mu_x^4 \qquad 2.8$$

When comparing measures of the mean, standard deviation, skewness and kurtosis, the above quantities are standardized, or non-dimensionalized as follows:

$$\gamma = \frac{\sigma_x}{\mu_x^{(1)}} \qquad 2.9$$

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_x^{(3)}}{\sigma_x^3} = \frac{\mu_x^{(3)}}{\left(\mu_x^{(2)}\right)^{3/2}} = \frac{E\left[x - \mu_x^{(1)}\right]^3}{\left(E\left[x - \mu_x^{(1)}\right]^2\right)^{3/2}} \qquad 2.10$$

$$\gamma_2 = \beta_2 - 3 = \frac{\mu_x^{(4)}}{\sigma_x^4} - 3 = \frac{\mu_x^{(4)}}{\left(\mu_x^{(2)}\right)^2} - 3 = \frac{E\left[x - \mu_x^{(1)}\right]^4}{\left(E\left[x - \mu_x^{(1)}\right]^2\right)^2} - 3 \qquad 2.11$$

Notation and terminology vary widely depending on the reference used. Here, γ is the coefficient of variation, γ_1 is the measure of skewness, and γ_2 is the measure of kurtosis. However, γ_1 and γ_2 can be referred to as the third and forth cumulants, or shape factors as well. [Ref. 4 and Ref. 5] The -3 is a centralizing term used when comparing measures of kurtosis. It comes from the fact that the kurtosis for a Normal distribution is 3. Some references use this correction, others do not. In this thesis the correction was **not** used. Hence, $\gamma_2 = \beta_2 = \frac{\mu_x^{(4)}}{\sigma_x^4} = \ldots.$

The underlying approach of the distribution moments method is that if moment values of sample data from an unknown population match the moments of a standard distribution, it would be the appropriate distribution with which to characterize the data. This is done by plotting the moments calculated from the data over a template of moment functions calculated for various standard distributions. If the sample moment values fall on, or near, a known distribution curve, it could indicate that the sample's population distribution corresponds. The distributions treated in this moment method are the Normal, Lognormal, Exponential, and Weibull. Each of these distributions has a unique probability density function that can be integrated and used to derive a function for each of the moments defined above. Once the moment functions are derived, they can be plotted versus one another, and a template of curves can be formed.

B. DERIVATION OF MOMENT FUNCTIONS

1. Normal Distribution

Equation 2.12 is the probability density function (pdf) for the Normal distribution.

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right], \quad -\infty < x < \infty \qquad 2.12$$

The x subscript has been dropped from the mean and standard deviation without creating any ambiguity. The pdf is substituted into Equation 2.1 and a general expression for the moments is developed.

$$\mu^{(r)} = E\left[(X-\mu)^r\right] = \int_{-\infty}^{\infty} \frac{(x-\mu)^r}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] dx \qquad 2.13$$

To simplify this integral a variable change is needed to the standard Normal z. Let $z=\frac{(x-\mu)}{\sigma} \Rightarrow x=z\sigma+\mu$; then Equation 2.13 becomes:

$$\mu^{(r)}=\sigma^{r}\int_{-\infty}^{\infty}\frac{z^{r}}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}z^{2}\right]dz \qquad 2.14$$

For odd integer values of r the integral is zero, but for even integer values of r, Equation 2.14 can be written as:

$$\mu^{r}=\sigma^{r}\int_{0}^{\infty}\frac{2z^{r}}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}z^{2}\right]dz \qquad 2.15$$

This is a variation of Euler's Second integral where:

$$2^{S-1}\Gamma(s)=\int_{0}^{\infty}z^{2S-1}\exp\left[-\frac{1}{2}z^{2}\right]dz \qquad 2.16$$

Letting $s=\frac{1}{2}(r+1)$, and substituting into Equation 2.15 yields: [Ref. 3]

$$\mu^{(r)}=\frac{\sigma^{r}2^{r/2}\Gamma\left(\frac{1}{2}(r+1)\right)}{\sqrt{\pi}} \qquad 2.17$$

Since odd integer values of r result in $\mu^{(r)}=0$, the standardized measure for skewness, Equation 2.10 becomes:

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_x^{(3)}}{\sigma_x^3} = \frac{\mu_x^{(3)}}{\left(\mu_x^{(2)}\right)^{3/2}} = 0 \qquad 2.18$$

which indicates a symmetrical distribution. Using properties of the gamma function, the values of $\mu^{(2)}$ and $\mu^{(4)}$ are:

$$\mu^{(2)} = \frac{\sigma^2 2\, \Gamma\left(\frac{3}{2}\right)}{\sqrt{\pi}} = \sigma^2 \qquad 2.19$$

$$\mu^{(4)} = \frac{\sigma^4 2^2 \Gamma\left(\frac{5}{2}\right)}{\sqrt{\pi}} = 3\sigma^4 \qquad 2.20$$

Substituting these values into Equation 2.11 for the standardized measure of kurtosis yields:

$$\gamma_2 = \beta_2 = \frac{\mu_x^{(4)}}{\sigma_x^4} = \frac{\mu_x^{(4)}}{\left(\mu_x^{(2)}\right)^2} = \frac{3\sigma^4}{\sigma^4} = 3 \qquad 2.21$$

Recall, the correction factor of -3 is not being used.

2. Lognormal Distribution

Using the random variable Z, where $x = \ln z$ is normal and the constants ξ and δ as parameters, where $\mu_x = \xi$ and $\sigma_x = \delta$, the Lognormal probability density function for Z becomes:

$$f_z(z) = \frac{1}{\sqrt{2\pi\, z^2\delta^2}} \exp\left[\frac{-1}{2\delta^2}(\ln z - \xi)^2\right], \quad \text{for } z \geq 0$$
$$= 0, \quad \text{for } z < 0 \tag{2.22}$$

Substituting this equation into Equation 2.1 yields a general form for the expected value of the rth moment of Z.

$$E[Z^r] = \int_0^\infty \frac{z^{r-1}}{\sqrt{2\pi\delta^2}} \exp\left[\frac{-1}{2\delta^2}(\ln z - \xi)^2\right] dz \tag{2.23}$$

By changing the variable to $y = \ln z - \xi$; then $z = e^\xi e^y$, and $dz = e^\xi e^y dy$, and $-\infty < y < \infty$ [Ref. 3]. Substituting and simplifying:

$$E\left[Z^r\right] = \int_{-\infty}^{\infty} \frac{\left(e^y e^\xi\right)^{r-1}}{\sqrt{2\pi\delta^2}} \exp\left[\frac{-1}{2\delta^2}\left(\ln\left(e^y e^\xi\right) - \xi\right)^2\right] e^y e^\xi dy \tag{2.24}$$

$$= \int_{-\infty}^{\infty} \frac{e^{yr} e^{\xi r}}{\sqrt{2\pi\delta^2}} \exp\left[\frac{-y^2}{2\delta^2}\right] dy \tag{2.25}$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi\delta^2} \exp\left[yr + \xi r - \frac{y^2}{2\delta^2}\right] dy \tag{2.26}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\delta^2}} \frac{e^{r\xi} e^{ry}}{\exp\left(\frac{y^2}{2\delta^2}\right)} dy \tag{2.27}$$

$$= e^{r\xi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\delta^2}} \exp\left[\frac{-1}{2\delta^2}\left(-2\delta^2 ry + y^2\right)\right] dy \tag{2.28}$$

At this point $\delta^4 r^2$ is added and subtracted to the exponential to make $\left(-2\delta^2 ry + y^2\right)$ a perfect square.

$$E\left[Z^r\right] = e^{r\xi} e^{\left(\frac{\delta^4 r^2}{2\delta^2}\right)} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[\frac{-1}{2\delta^2}\left(y^2 - 2\delta^2 ry + \delta^4 r^2\right)\right] dy \qquad 2.29$$

Finally:

$$E[Z^r] = \exp\left[r\,\xi + \frac{1}{2}\delta^2 r^2\right] \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[\frac{-1}{2\delta^2}\left(y - \delta^2 r\right)^2\right] dy \qquad 2.30$$

The integral factor is the same as the cumulative density function for the Normal distribution, which is normalized to one. Therefore, the general expression for the expectance is:

$$E[Z^r] = \exp\left[r\,\xi + \frac{1}{2}\delta^2 r^2\right] \qquad 2.31$$

and,

$$\mu = E\left[Z\right] = \exp\left[\ \xi + \frac{1}{2}\delta^2\right] \qquad 2.32$$

The coefficient of variation from Equation 2.9 is:

$$\gamma = \left(\exp\delta^2 - 1\right)^{1/2} \qquad 2.32a$$

However, unlike the Normal distribution where a general form for $\mu^{(r)}$ was derived, here each moment must be expanded algebraically and substituted into Equations 2.10 and 2.11. For example, to calculate the measure of skewness (γ_1), $\mu^{(2)}$

and $\mu^{(3)}$ must first be calculated. The necessary substitutions with some examples of the algebra are shown below. Equation 2.4, $\mu^{(2)} = E\left[X^2\right] - 2\mu E\left[X\right] + (\mu)^2$ becomes:

$$\mu^{(2)} = \exp\left[2\xi + 2\delta^2\right] - 2\exp\left[2\xi + \delta^2\right] + \exp\left[2\xi + \delta^2\right] = \exp\left[2\xi + 2\delta^2\right] - \exp\left[2\xi + \delta^2\right] \qquad 2.33$$

With a similar substitution, Equation 2.6, for the third moment $\mu_x^{(3)} = E\left[X^3\right] - 3\mu_x E\left[X^2\right] + 3\mu_x^2 E\left[X\right] - \mu_x^3$, becomes:

$$\mu^{(3)} = \exp\left[3\xi + \frac{9}{2}\delta^2\right] - 3\exp\left[\xi + \frac{1}{2}\delta^2\right]\exp\left[2\xi + 2\delta^2\right] + 3\exp\left[2\xi + \delta^2\right]\exp\left[\xi + \frac{1}{2}\delta^2\right] - \exp\left[3\xi + \frac{3}{2}\delta^2\right]$$

$$\begin{aligned} &= \exp\left[3\xi + \frac{9}{2}\delta^2\right] - 3\exp\left[3\xi + \frac{5}{2}\delta^2\right] + 3\exp\left[3\xi + \frac{3}{2}\delta^2\right] - \exp\left[3\xi + \frac{3}{2}\delta^2\right] \\ &= \exp\left[3\xi + \frac{9}{2}\delta^2\right] - 3\exp\left[3\xi + \frac{5}{2}\delta^2\right] + 2\exp\left[3\xi + \frac{3}{2}\delta^2\right] \end{aligned} \qquad 2.34$$

Note that $E\left[X^3\right] \neq \mu^3$.

The measure of skewness, Equation 2.10 simplifies to:

$$\gamma_1 = \frac{\exp\left[3\delta^2\right] - 3\exp\left[\delta^2\right] + 2}{\left(\exp\left[\delta^2\right] - 1\right)^{3/2}} \qquad 2.35$$

The measure of kurtosis γ_2, can be derived once the expression for $\mu^{(4)}$ is expanded and simplified. The algebra is too extensive to show. Only the simplified function is given.

$$\gamma_2 = \exp\left[4\delta^2\right] + 2\exp\left[3\delta^2\right] + 3\exp\left[2\delta^2\right] - 3 \qquad 2.36$$

The functions in Equations 2.32a, 2.35, and 2.36 show that the coefficient of variation and the measures of skewness and kurtosis depend only on the value of δ, the standard deviation of the population.

3. Exponential Distribution

The probability density function for the exponential distribution is:

$$f_x(x) = \theta \exp(-\theta x) \quad , \text{ if } x \geq 0 \qquad 2.37$$

Substituting into Equation 2.1a yields the general expression for the expectance:

$$E[X^r] = \int_0^\infty x^r \left(\theta \exp(-\theta x)\right) dx \qquad 2.38$$

Performing the variable substitution: $v = \theta x \Rightarrow x = \frac{v}{\theta}$ and $dv = \theta dx$,

The expectance becomes:

$$E\left[X^r\right] = \int_0^\infty \left(\frac{v}{\theta}\right)^r e^{-v} dv = \frac{1}{\theta^r} \int_0^\infty v^r e^{-v} dv \qquad 2.39$$

This integral has the same form as Euler's Second Integral, called the gamma function [Ref. 3].

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \qquad 2.40$$

Substituting v for t, and letting $r+1=z$, the simplified expression for the expectance is:

$$E[X^r]=\frac{\Gamma(r+1)}{\theta^r} \qquad 2.41$$

Using Equation 2.41 and properties of the gamma function, expressions for μ, $\mu^{(2)}$, $\mu^{(3)}$, and $\mu^{(4)}$ are shown below.

$$\mu=\frac{\Gamma(2)}{\theta}=\frac{1}{\theta} \qquad 2.42$$

$$\mu^{(2)}=\frac{\Gamma(3)}{\theta^2}-\frac{2\Gamma(2)}{\theta^2}+\frac{1}{\theta^2}=\frac{1}{\theta^2} \qquad 2.43$$

$$\mu^{(3)}=\frac{\Gamma(4)}{\theta^3}-\left(\frac{3}{\theta}\right)\left(\frac{\Gamma(3)}{\theta^2}\right)+\left(\frac{3}{\theta^2}\right)\left(\frac{1}{\theta}\right)-\frac{1}{\theta}=\frac{2}{\theta^3} \qquad 2.44$$

$$\mu^{(4)}=\frac{\Gamma(5)}{\theta^4}-\left(\frac{4}{\theta}\right)\left(\frac{\Gamma(4)}{\theta^3}\right)+\left(\frac{6}{\theta^2}\right)\left(\frac{\Gamma(3)}{\theta^2}\right)-\left(\frac{4}{\theta^3}\right)\left(\frac{\Gamma(2)}{\theta}\right)+\frac{1}{\theta^4}=\frac{9}{\theta^4} \qquad 2.45$$

Constructing the numerators and denominators for Equations 2.10 and 2.11, using the results of Equations 2.42 through 2.45, produces a coefficient of variation of $\gamma=1$, a measure of skewness of $\gamma_1=2$, and a measure of kurtosis of $\gamma_2=9$. These will plot as points on the template.

4. Weibull Distribution

The parameters of the Weibull distribution are, β which determines the shape, $1/\alpha$ which plays the same role as θ did in the exponential distribution, and ν which is the smallest value the random variable can assume. The probability density function for the Weibull distribution is:

$$f_x = \frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], \text{ if } x \geq \nu \qquad 2.46$$

$$= 0 \ , \text{ if } x < \nu$$

The Weibull and Exponential distributions are related by the fact that if β=1, ν=0, and $\theta=1/\alpha$ the two are identical. Therefore if X has a Weibull distribution with parameters β, α, and ν; then $Y = \left[\frac{(X-\nu)}{\alpha}\right]^{\beta}$ has an Exponential distribution with parameter θ=1. Or, $X = \alpha Y^{\frac{1}{\beta}} + \nu$, where X and $\alpha Y^{\frac{1}{\beta}} + \nu$ have the same distributions, means, variances, and higher moments. Now, where $E[X^r] = \frac{\Gamma(r-1)}{\theta^r}$, and $\mu = \frac{1}{\theta}$ for the Exponential distribution,

$$E[X^r] = E\left[\left(\alpha Y^{\frac{1}{\beta}} + \nu\right)^r\right] \qquad 2.47a$$

$$\mu = E[X] = E\left[\left(\alpha Y^{1/\beta} + \nu\right)\right] = \alpha E\left[Y^{1/\beta}\right] + \nu = \frac{\alpha\Gamma\left(\frac{1}{\beta}+1\right)+\nu}{\theta} \qquad 2.47b$$

for the Weibull distribution. Since θ must equal one for the similarity to work, $\mu = \alpha\Gamma\left(\frac{1}{\beta}+1\right)+\nu$.

Letting ν=0, these expressions are used to construct the moments used in Equations 2.10 and 2.11. The algebra is too involved to show here, but the substitutions are made just as they were for the Exponential case in Equations 2.42 through 2.45. After algebraically reducing the expressions, functions for the coefficient of variation (γ), measures of skewness (γ_1) and kurtosis (γ_2) are shown below.

$$\gamma = \frac{\left[\Gamma\left(\frac{2}{\beta}+1\right)-\Gamma^2\left(\frac{1}{\beta}+1\right)\right]^{1/2}}{\Gamma\left(\frac{1}{\beta}+1\right)} \tag{2.48}$$

$$\gamma_1 = \frac{2\Gamma^3\left(\frac{1}{\beta}+1\right)-3\Gamma\left(\frac{2}{\beta}+1\right)\Gamma\left(\frac{1}{\beta}+1\right)+\Gamma\left(\frac{3}{\beta}+1\right)}{\left[\Gamma\left(\frac{2}{\beta}+1\right)-\Gamma^2\left(\frac{1}{\beta}+1\right)\right]^{3/2}} \tag{2.49}$$

$$\gamma_2 = \frac{\Gamma\left(\frac{4}{\beta}+1\right)-4\Gamma\left(\frac{3}{\beta}+1\right)\Gamma\left(\frac{1}{\beta}+1\right)+6\Gamma\left(\frac{2}{\beta}+1\right)\Gamma^2\left(\frac{1}{\beta}+1\right)-3\Gamma^4\left(\frac{1}{\beta}+1\right)}{\Gamma^2\left(\frac{2}{\beta}+1\right)-2\Gamma\left(\frac{2}{\beta}+1\right)\Gamma^2\left(\frac{1}{\beta}+1\right)+\Gamma^4\left(\frac{1}{\beta}+1\right)} \tag{2.50}$$

These expressions show that the measures of skewness and kurtosis for a Weibull distribution depend only on the value of β, the shape parameter of the distribution.

Once the moment functions were derived, they were entered on a spread sheet in Microsoft Excel, where values of the coefficient of variation, skewness, and kurtosis could be calculated for several different values of the distribution parameters. Three plots were constructed: (1) skewness versus coefficient of variation, (2) kurtosis versus skewness, and (3) kurtosis versus coefficient of variation. Figures 2.1, 2.2, and 2.3 show the three plots with the curves for each distribution. Notice that the Normal and Exponential distributions appear as single points on some of the plots.

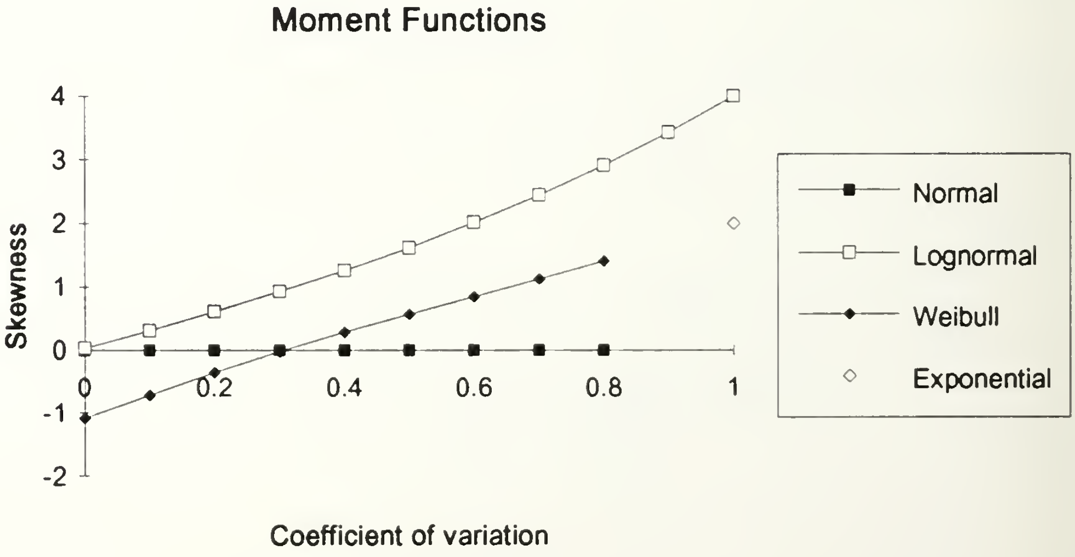


Figure 2.1: Moment Functions

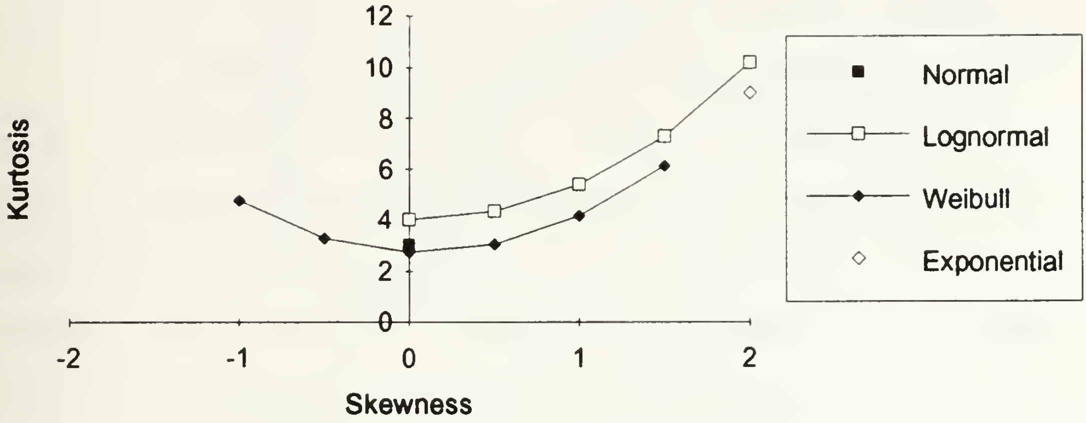


Figure 2.2: Moment Functions

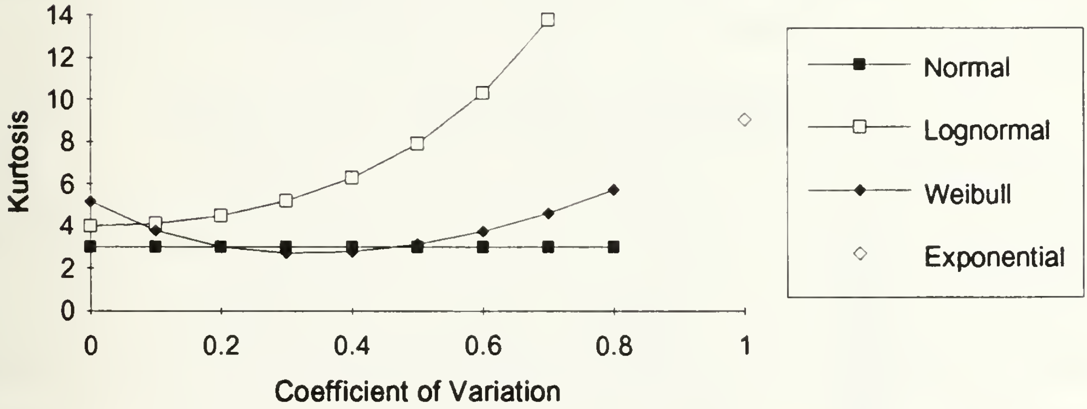


Figure 2.3: Moment Functions

C. TESTING OF THE MOMENT PLOT METHOD

1. Random Number Draw

a. A Graphical Statistical System (AGSS)

To test whether or not the method would discriminate between sample population distributions, random numbers drawn from known distributions were analyzed using the moment plot method. A Graphical Statistical System (AGSS), an APL based software package resident on the Naval Postgraduate School's main-frame computer, was used to generate the random numbers, and to calculate the coefficients of variation, and the measures of skewness and kurtosis of the samples analyzed. As part of the analysis procedure, AGSS makes histograms, cumulative distribution plots, and probability plots for each distribution it fits to data. Figures 2.4a, b, and c are examples of these plots.

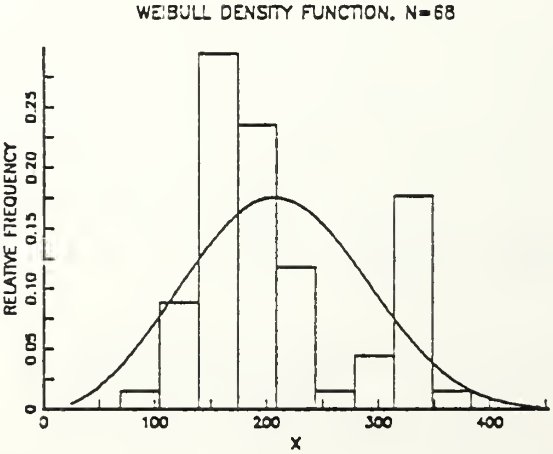


Figure 2.4a: Histogram and Weibull Density Function

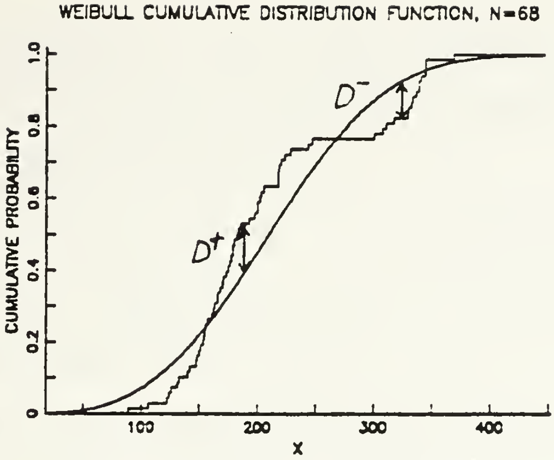


Figure 2.4b: EDF and Weibull CDF

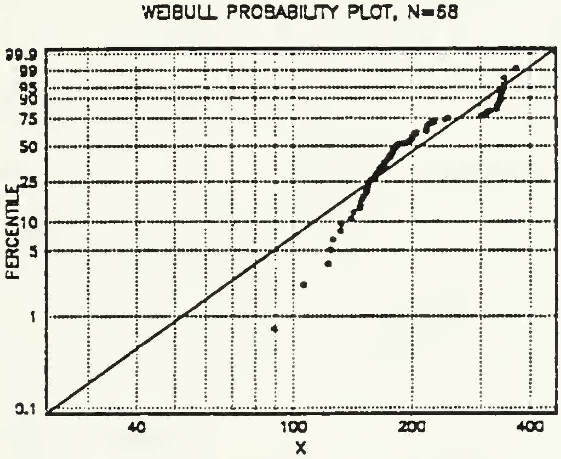


Figure 2.4c: Weibull Probability Plot

Figure 2.4a is a histogram of the sample values with the Weibull density function. Comparing the two, shows how the frequencies of sample values (represented by the bars) differ from the expected frequencies of values from a Weibull distribution density function. The empirical cumulative distribution function (EDF) is shown in Figure 2.4b with the Weibull cumulative distribution function (CDF). If this sample had a perfect Weibull distribution the empirical plot would lie on top of the Weibull function. Figure 2.4c is a Weibull probability plot. It conveys the same information as Figure 2.4b, but is used to more easily judge the goodness of fit. If the sample had a perfect Weibull distribution, the points plotted would lie on the straight line. To achieve the straight line for probability; y-axis has been plotted as $y = \ln(-\ln(1 - F_n(x)))$, and the x-axis becomes $\ln(x)$. Where $F_n(x)$ is the Weibull cumulative distribution function [Ref. 6].

b. Types of Data to be Fitted

To generate random numbers from a specific population distribution, AGSS asks for certain input parameters. For Normal, and Lognormal distributions, the mean and standard deviation are required. For Weibull distributions, the shape and scale parameters are required, and for Exponential distributions the mean is required. The range and variance of typical flight load data and fatigue life data were used to determine the input parameters. Two means were chosen to represent the loads data, 500 lbs., and 5000 lbs. Three means were chosen to represent the fatigue life data, 100,000, 1,000,000, and 5,000,000 cycles. Standard deviations typical of the data were chosen for these means, which were 150, 1500, 30,000, 150,000, and 750,000, respectively. Five separate random number draws corresponding to these means and standard deviation pairs were made from each of the four distributions.

c. Distribution Fitting

Once the 20 samples of random numbers were generated, each sample was fit to each of the four distributions. Figures 2.5a, b, c, d, e, f, g, and h show the CDF/EDF plots and probability plots for a Weibull sample fit to the four different distributions.

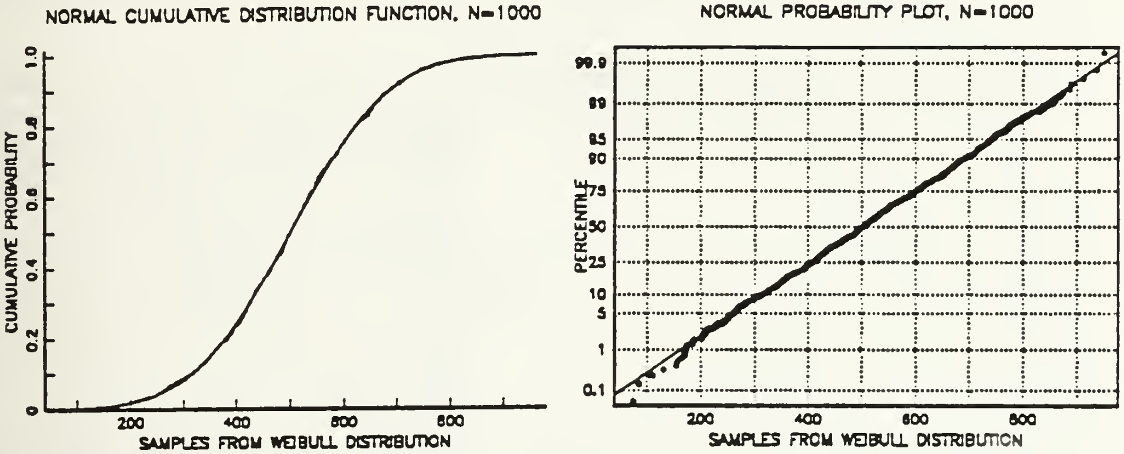


Figure 2.5a: EDF/CDF **Figure 2.5b: Probability**
Weibull Sample fit to Normal Distribution

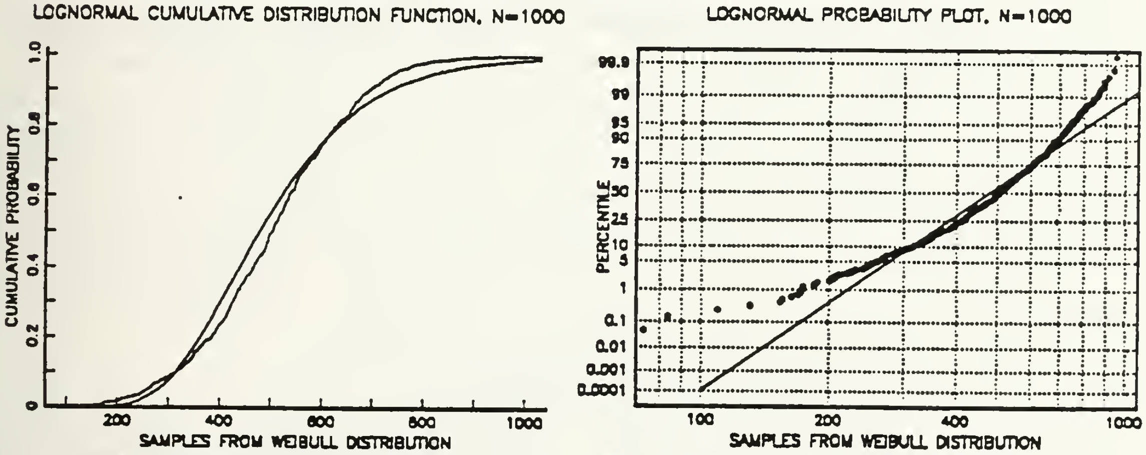


Figure 2.5c: EDF/CDF **Figure 2.5d: Probability**
Weibull Sample fit to Lognormal Distribution

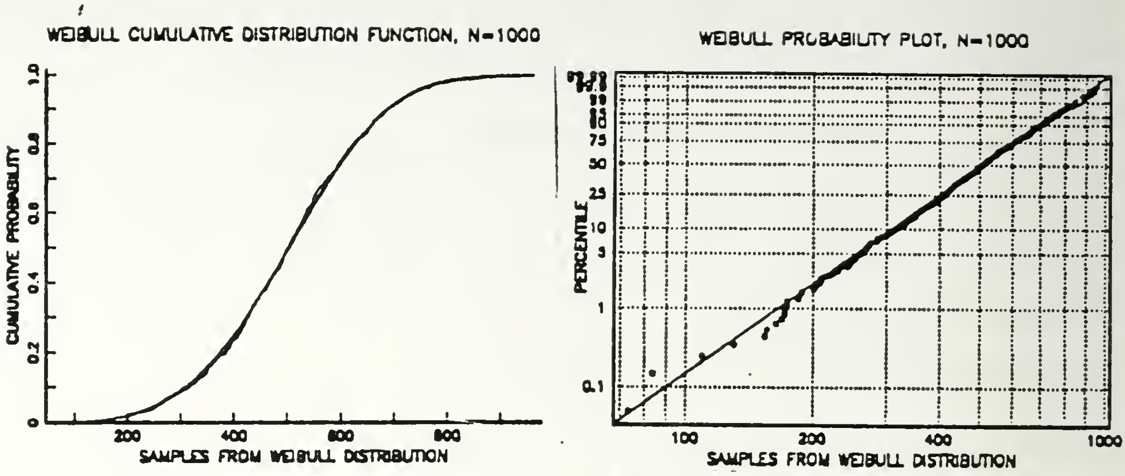


Figure 2.5e: EDF/CDF Figure 2.5f: Probability
Weibull Sample fit to Weibull Distribution

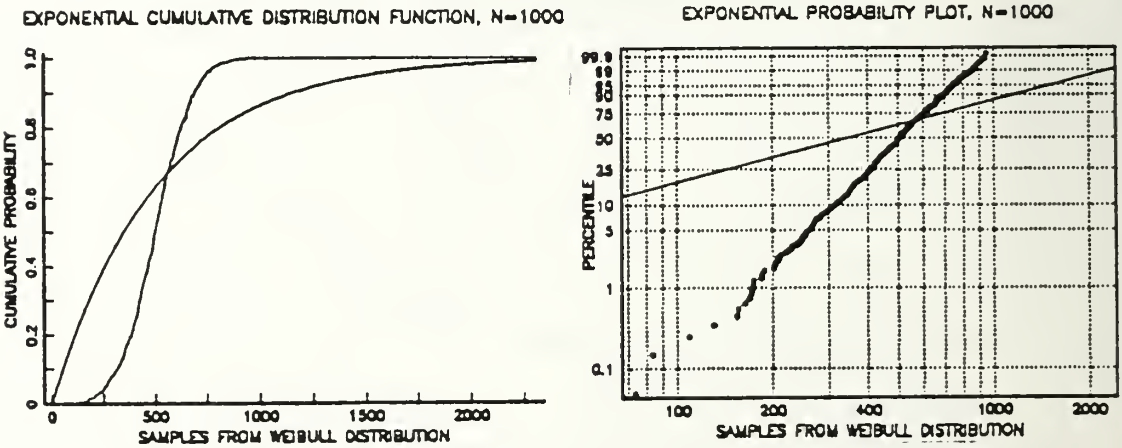


Figure 2.5g: EDF/CDF Figure 2.5h: Probability
Weibull Sample fit to Exponential Distribution

AGSS calculated the sample mean, standard deviation, skewness, and kurtosis for each sample (20 sets of values). It also calculated four goodness of fit measures for each fit (80 sets of values).

d. Goodness of Fit Measures

AGSS calculates the Chi-square, Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness of fit statistics. The Chi-square goodness of fit test is applicable to discrete or continuous, univariate or multivariate data, and is the most widely used goodness of fit test. There are several variations of the test, all based on the following relation, which is a measure of the difference between the observed and the expected frequency of a value. A graphical representation of this difference is Figure 2.4a.

$$X^2 = \sum_{i=1}^{M} \frac{(N_i - np_i)^2}{np_i} \qquad 2.51$$

Here, N is the observed frequency of the ith cell. The expected frequency is np_i based on the probability density. The other three goodness of fit statistics are "EDF statistics", based on measuring the vertical difference between the empirical distribution function of the sample, $F_n(x)$, and the cumulative distribution function of the distribution being tested, $F(x)$. Figure 2.4b shows the difference between $F_n(x)$ and $F(x)$. These vertical differences are divided into two classes, the supremum class and the quadratic class.

The supremum statistics are shown in Figure 2.4b, where D^+ is the largest vertical difference when $F_n(x)$ is greater than $F(x)$, and D^- is the largest vertical difference when $F_n(x)$ is smaller than $F(x)$. The most well-known EDF statistic is the Kolmogorov statistic D. It is defined as: $D = \sup_x |F_n(x) - F(x)|$, and measures the maximum difference between $F_n(x)$, and $F(x)$. [Ref. 5 and Ref. 6]

The quadratic class is given by:

$$Q = n\int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi(x)\, dF(x) \qquad 2.52$$

where $\psi(x)$ is a weighting function for the squared difference $\{F_n(x) - F(x)\}^2$. The Cramer-von Mises statistic W^2, is a special case of the quadratic class where $\psi(x) = 1$. For the Cramer-von Mises statistic, Equation 2.52 specializes to:

$$W^2 = n\int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 dF(x) \qquad 2.53$$

The Anderson-Darling statistic A^2, is another special case of the quadratic statistic where $\psi(x) = \left[\{F(x)\}\{(1 - F(x))\}\right]^{-1}$. For the Anderson-Darling statistic, Equation 2.53 specializes to:

$$A^2 = n\int_{-\infty}^{\infty} \frac{\{F_n(x) - F(x)\}^2 dF(x)}{\{F(x)\}\{(1 - F(x))\}} dF(x) \qquad 2.54$$

The quadratic statistics are more powerful than the supremum class of EDF statistics, since better use is made of the information contained in the whole sample rather than by using only the maximum discrepancy [Ref. 6]. Smaller values of the four goodness of fit statistics indicate a better fit to the tested distribution.

2. Results of the Study

a. Moment Plots

The following three figures show moments of the random samples plotted on the moment function templates. There are 20 points in each figure representing the five random draws from each of the four distributions.

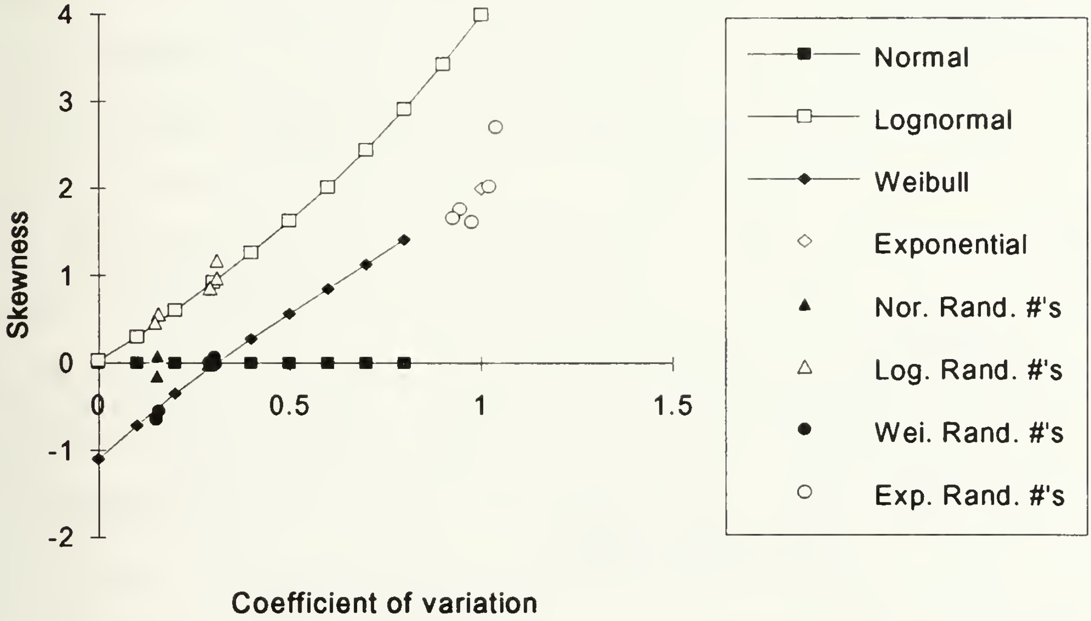


Figure 2.6: Coefficient of Variation vs. Skewness

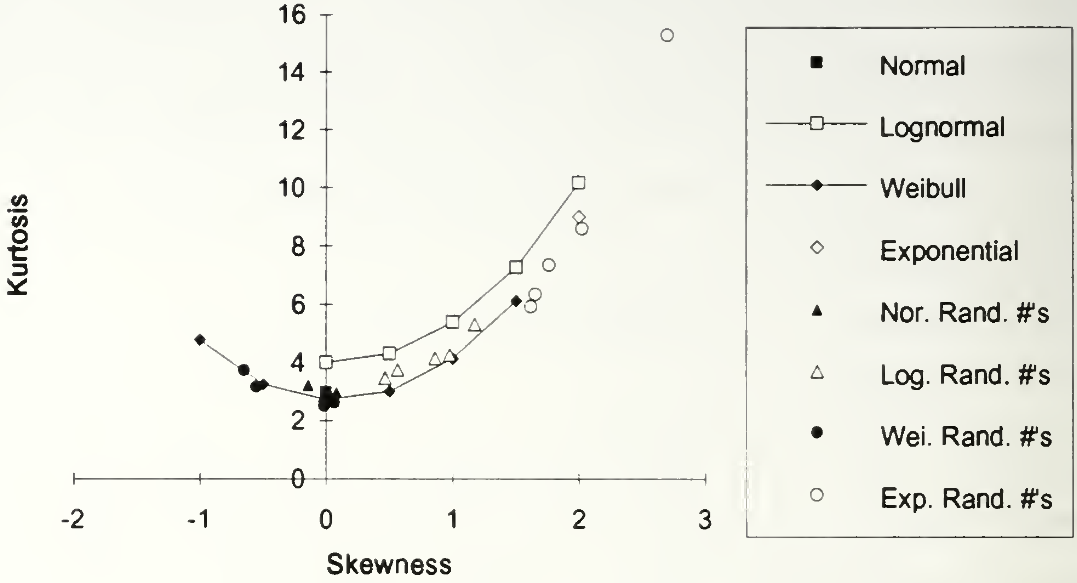


Figure 2.7: Kurtosis vs. Skewness

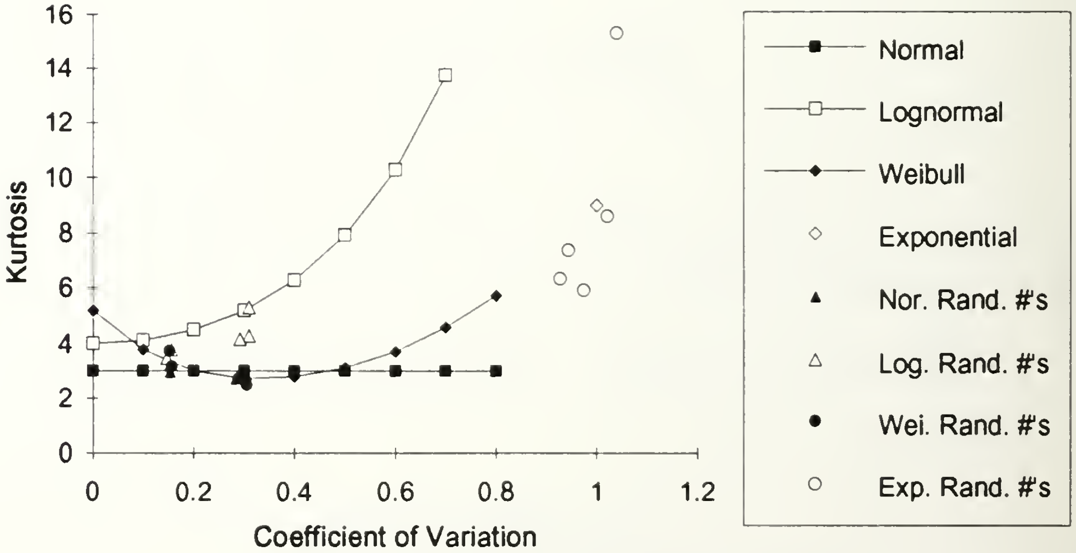


Figure 2.8: Kurtosis vs. Coefficient of Variation

The values of skewness and the coefficients of variation for the random samples plotted in Figure 2.6 fall close to their correct distribution curves. Where the Normal and Weibull functions intersect at (.25,0) there are three points from the Normal random sample and three points from the Weibull random sample. This intersection is the only area of ambiguity for this particular plot. Aside from this area, the plot indicates the correct distributions of the random samples. The random sample values of skewness and kurtosis (Figure 2.7) all fall near the curve for a Weibull distribution, but the kurtosis is consistently underestimated for Lognormal. This plot may not be as useful an indicator of the distribution as Figure 2.6. Figure 2.8 has three intersection points that could result in ambiguous distribution indications. By using all three plots, it was hoped that the areas of ambiguity could be resolved.

b. Goodness of Fit Tables

Goodness of fit statistics were calculated for each fit of the random samples and entered in the following tables. Significance levels are also listed under their respective goodness of fit statistic. AGSS specifies a significance level $\alpha = 0.01$ for the goodness of fit statistics. It uses as the null hypothesis, H_o: "The goodness of fit statistic is small enough to indicate a good fit". For each goodness of fit statistic, AGSS calculates the corresponding p-value from the sample data. If $p \leq \alpha$, the null hypothesis must be rejected [Ref. 7]. Therefore, $p \leq 0.01$ indicates a lack of fit. Tables 2.1 through 2.5 are goodness of fit statistics for samples drawn from a Normal distribution, then fit to all four distributions. Tables 2.6 through 2.10 are for samples drawn from a Lognormal distribution. Tables 2.11 through 2.15 are for samples from a Weibull distribution, and Tables 2.16 through 2.20 are for samples drawn from an Exponential distribution.

TABLE 2.1: NORMAL SAMPLE 1

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	**12.508**	114.04	7.9631	1674.2
Significance	**0.12993**	0	0.33585	0
Kolm-Smirn	**0.02433**	0.077617	0.016414	0.37174
Significance	**0.59467**	0.000011701	0.95044	0
Cramer-V M	**0.067764**	2.3193	0.035885	47.941
Significance	**>0.15**	<0.01	>0.15	<0.01
Ander-Darl	**0.42392**	infinite	0.34355	233.6
Significance	**>0.15**	0	>0.15	<0.01

TABLE 2.2: NORMAL SAMPLE 2

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	**5.8328**	111	6.0472	1611
Significance	**0.66595**	0	0.64194	0
Kolm-Smirn	**0.013458**	0.080474	0.022106	0.36275
Significance	**0.99351**	0.0000047422	0.7128	0
Cramer-V M	**0.022671**	1.8381	0.062938	46.724
Significance	**>0.15**	<0.01	>0.15	<0.01
Ander-Darl	**0.16054**	11.45	0.35882	228.21
Significance	**>0.15**	<0.01	>0.15	<0.01

TABLE 2.3: NORMAL SAMPLE 3

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	**2.7318**	103.09	5.4523	1654.6
Significance	**0.95004**	0	0.60494	0
Kolm-Smirn	**0.018078**	0.070662	0.01623	0.3677
Significance	**0.89943**	0.000092067	0.95485	0
Cramer-V M	**0.031066**	1.8326	0.047165	46.978
Significance	**>0.15**	<0.01	>0.15	<0.01
Ander-Darl	**0.19491**	11.707	0.3155	229.1
Significance	**>0.15**	<0.01	>0.15	<0.01

TABLE 2.4: NORMAL SAMPLE 4

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	**7.6794**	40.694	32.709	4144
Significance	**0.36171**	9.267E-7	0.000069474	0
Kolm-Smirn	**0.017169**	0.046771	0.04403	0.48245
Significance	**0.92973**	0.025173	0.041417	0
Cramer-V M	**0.046794**	0.58789	0.49186	71.241
Significance	**>0.15**	<0.025	<0.05	<0.01
Ander-Darl	**0.37196**	4.0338	3.1213	333.17
Significance	**>0.15**	<0.01	<0.025	<0.01

TABLE 2.5: NORMAL SAMPLE 5

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	**8.1368**	23.493	38.053	3885
Significance	**0.32068**	0.0013981	0.0000029604	0
Kolm-Smirn	**0.020689**	0.033402	0.04478	0.48708
Significance	**0.78541**	0.2145	0.036249	0
Cramer-V M	**0.0471**	0.27747	0.54246	70.769
Significance	**>0.15**	>0.15	<0.05	<0.01
Ander-Darl	**0.32459**	1.8754	4.0146	331.03
Significance	**>0.15**	<0.15	<0.01	<0.01

TABLE 2.6: LOGNORMAL SAMPLE 1

Test\Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	129.25	**5.1854**	127.1	1538.6
Significance	0	**0.63735**	0	0
Kolm-Smirn	0.078013	**0.021541**	0.083086	0.3987
Significance	0	**0.74231**	0	0
Cramer-V M	1.8897	**0.06254**	2.175	47.983
Significance	<0.01	**>0.15**	<0.01	<0.01
Ander-Darl	11.906	**0.47427**	14.545	232.17
Significance	<0.01	**>0.15**	<0.01	<0.01

TABLE 2.7: LOGNORMAL SAMPLE 2

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	93.243	**3.8938**	104.78	1690.6
Significance	0	**0.79192**	0	0
Kolm-Smirn	0.076753	**0.020737**	0.07986	0.40182
Significance	0	**0.78304**	0	0
Cramer-V M	1.3541	**0.059057**	1.7924	49.96
Significance	<0.01	**>0.15**	<0.01	<0.01
Ander-Darl	8.2654	**0.31629**	11.959	241
Significance	<0.01	**>0.15**	<0.01	<0.01

TABLE 2.8: LOGNORMAL SAMPLE 3

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	159.15	**9.989**	153.32	1654.4
Significance	0	**0.12512**	0	0
Kolm-Smirn	0.087918	**0.029511**	0.089763	0.40361
Significance	3.8656E-7	**0.34853**	2.0068E-7	0
Cramer-V M	2.2149	**0.13525**	2.8142	49.108
Significance	<0.01	**>0.15**	<0.01	<0.01
Ander-Darl	13.612	**0.80713**	18.485	236.68
Significance	<0.01	**>0.15**	<0.01	<0.01

TABLE 2.9: LOGNORMAL SAMPLE 4

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	33.105	**4.4023**	121.07	4021
Significance	0	**0.6224**	0	0
Kolm-Smirn	0.051168	**0.021552**	0.076314	0.49442
Significance	0.010641	**0.74173**	0	0
Cramer-V M	0.64474	**0.076439**	2.4009	71.333
Significance	<0.025	**>0.15**	<0.01	<0.01
Ander-Darl	3.8662	**0.50689**	15.696	332.8
Significance	<0.01	**>0.15**	<0.01	<0.01

TABLE 2.10: LOGNORMAL SAMPLE 5

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	32.089	**4.5198**	130.89	4311.2
Significance	0	**0.71832**	0	0
Kolm-Smirn	0.047072	**0.023323**	0.080214	0.49845
Significance	0.023791	**0.64815**	0	0
Cramer-V M	0.43642	**0.076196**	2.0429	72.724
Significance	<0.01	**>0.15**	<0.01	<0.01
Ander-Darl	2.5507	**0.38184**	13.3	338.83
Significance	<0.05	**>0.15**	<0.01	<0.01

TABLE 2.11: WEIBULL SAMPLE 1

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	11.129	111.45	**7.9309**	1667.7
Significance	0.26697	0	**0.44025**	0
Kolm-Smirn	0.026625	0.065715	**0.022782**	0.37167
Significance	0.47761	0.00035486	**0.67701**	0
Cramer-V M	0.10739	1.502	**0.068919**	47.135
Significance	>0.15	<0.01	**>0.15**	<0.01
Ander-Darl	0.58359	9.7349	**0.39178**	230.33
Significance	>0.15	<0.01	**>0.15**	<0.01

TABLE 2.12: WEIBULL 2

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	13.324	95.08	**9.1728**	1490.8
Significance	0.1485	4.1633E-17	**0.32793**	0
Kolm-Smirn	0.02736	0.056852	**0.025149**	0.36073
Significance	0.44251	0.0031162	**0.55185**	0
Cramer-V M	0.13878	1.2526	**0.09691**	44.762
Significance	>0.15	<0.01	**>0.15**	<0.01
Ander-Darl	0.81389	8.2763	**0.56915**	219.82
Significance	>0.15	<0.01	**>0.15**	<0.01

TABLE 2.13: WEIBULL SAMPLE 3

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	15.197	128.99	**8.3147**	1474.9
Significance	0.085657	0	**0.50276**	0
Kolm-Smirn	0.025063	0.072904	**0.019056**	0.35731
Significance	0.55628	0.000048364	**0.86081**	0
Cramer-V M	0.13297	1.7007	**0.065984**	44.207
Significance	>0.15	<0.01	**>0.15**	<0.01
Ander-Darl	0.88182	10.695	**0.43388**	217.8
Significance	>0.15	<0.01	**>0.15**	<0.01

TABLE 2.14: WEIBULL SAMPLE 4

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4123.7	3.8461	**173.08**	66.461
Significance	0	0.87073	**0**	2.4798E-11
Kolm-Smirn	0.46119	0.017421	**0.086403**	0.054543
Significance	0	0.92191	**6.5556E-7**	0.0052128
Cramer-V M	69.88	0.038804	**2.4168**	0.78342
Significance	<0.01	>0.15	**<0.01**	<0.01
Ander-Darl	328.09	0.26589	**14.804**	4.8264
Significance	<0.01	>0.15	**<0.01**	<0.01

TABLE 2.15: WEIBULL SAMPLE 5

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4336.9	3.3931	**116.44**	57.413
Significance	0	0.75813	**0**	1.5068E-10
Kolm-Smirn	0.47111	0.021057	**0.089704**	0.052975
Significance	0	0.767	**2.0498E-7**	0.0073019
Cramer-V M	71.261	0.057894	**2.7003**	0.88649
Significance	<0.01	>0.15	**<0.01**	<0.01
Ander-Darl	333.82	0.35343	**15.907**	5.2765
Significance	<0.01	>0.15	**<0.01**	<0.01

TABLE 16: EXPONENTIAL SAMPLE 1

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	109.61	88.212	6.2758	**6.3374**
Significance	0	6.9389E-17	0.17947	**0.27476**
Kolm-Smirn	0.16819	0.08944	0.02144	**0.022324**
Significance	5.3839E-25	2.2525E-7	0.74747	**0.70129**
Cramer-V M	8.2089	2.3331	0.059895	**0.065358**
Significance	<0.01	<0.01	>0.15	**>0.15**
Ander-Darl	48.045	13.812	0.39738	**0.42406**
Significance	<0.01	<0.01	>0.15	**>0.15**

TABLE 2.17: EXPONENTIAL SAMPLE 2

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	132.13	114.3	7.7534	**11.213**
Significance	0	0	0.25674	**0.12958**
Kolm-Smirn	0.1451	0.095011	0.023862	**0.034432**
Significance	1.0345E-18	2.8853E-8	0.61946	**0.1866**
Cramer-V M	6.3655	2.7706	0.086885	**0.21045**
Significance	<0.01	<0.01	>0.15	**>0.15**
Ander-Darl	38.399	16.461	0.51685	**0.98136**
Significance	<0.01	<0.01	>0.15	**>0.15**

TABLE 2.18: EXPONENTIAL SAMPLE 3

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	160.06	84.917	7.1068	**6.9939**
Significance	0	1.6945E-14	0.52516	**0.63775**
Kolm-Smirn	0.15239	0.072042	0.017363	**0.01737**
Significance	1.3516E-20	0.00006209	0.92377	**0.92353**
Cramer-V M	7.6478	1.7625	0.037868	**0.032433**
Significance	<0.01	<0.01	>0.15	**>0.15**
Ander-Darl	44.703	11.143	0.27649	**0.2461**
Significance	<0.01	<0.01	>0.15	**>0.15**

TABLE 2.19: EXPONENTIAL SAMPLE 4

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	210.62	72.891	6.9723	**8.9638**
Significance	0	1.3032E-12	0.43176	**0.34536**
Kolm-Smirn	0.14068	0.070094	0.017358	**0.044678**
Significance	1.2879E-17	0.00010801	0.9239	**0.036919**
Cramer-V M	7.1875	1.4596	0.058719	**0.30073**
Significance	<0.01	<0.01	>0.15	**<0.15**
Ander-Darl	41.998	9.5516	0.33573	**2.1475**
Significance	<0.01	<0.01	>0.15	**<0.1**

TABLE 2.20: EXPONENTIAL SAMPLE 5

Test\Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	151.03	70.603	9.1229	**9.5086**
Significance	0	3.0756E-13	0.10427	**0.14692**
Kolm-Smirn	0.16368	0.069234	0.017212	**0.017594**
Significance	1.076E-23	0.00013727	0.92842	**0.91629**
Cramer-V M	9.0392	1.8096	0.061076	**0.063071**
Significance	<0.01	<0.01	>0.15	**>0.15**
Ander-Darl	53.275	11.557	0.4929	**0.49039**
Significance	<0.01	<0.01	>0.15	**>0.15**

c. Vote Counting Method

The moment plots provide a method of visual comparison of the candidate population distribution moments with the sample moments, while the goodness of fit statistics provide an analytical comparison that is more abstract. The following vote count method combines the two procedures to produce a quantitative indication of the likely population distribution.

From the three moment plots (Figures 2.6, 2.7, 2.8), each of the 60 points provided a "vote" for a distribution, based on which distribution moment function the point was closest to. If the point fell equidistant between two functions, both corresponding distributions would receive a vote. The goodness of fit statistics "voted" for the distribution fit that produced the smallest statistic value; i.e., each table above produced four votes. The four tables below show the results of this vote counting method for each of the known distributions. The first three rows are the votes of the moment plots: the next four rows are the votes of the goodness of fit statistics.

TABLE 2.21: RANDOM SAMPLES FROM NORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
S. vs. C. of V.	5	0	3	0
Kurt. vs. Skew.	3	0	5	0
Kurt. vs. C. of V.	3	0	4	0
Chi-Square	4	0	1	0
Kolm.-Smirn.	3	0	2	0
Cramer-V.M.	4	0	1	0
Ander.-Dar.	4	0	1	0
Vote Totals:	26	0	17	0

TABLE 2.22: RANDOM SAMPLES FROM LOGNORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
S. vs. C. of V.	0	5	0	0
Kurt. vs. Skew.	0	0	5	0
Kurt. vs. C. of V.	0	3	2	0
Chi-Square	0	5	0	0
Kolm.-Smirn.	0	5	0	0
Cramer-V.M.	0	5	0	0
Ander.-Dar.	0	5	0	0
Vote Totals:	0	28	7	0

TABLE 2.23: RANDOM SAMPLES FROM WEIBULL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
S. vs. C. of V.	3	0	5	0
Kurt. vs. Skew.	0	0	5	0
Kurt. vs. C. of V.	0	0	5	0
Chi-Square	0	0	5	0
Kolm.-Smirn.	0	0	5	0
Cramer-V.M.	0	0	5	0
Ander.-Dar.	0	0	5	0
Vote Totals:	3	0	35	0

TABLE 2.24: RANDOM SAMPLES FROM EXPONENTIAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
S. vs. C. of V.	0	0	3	2
Kurt. vs. Skew.	0	0	4	1
Kurt. vs. C. of V.	0	0	4	1
Chi-Square	0	0	4	1
Kolm.-Smirn.	0	0	5	0
Cramer-V.M.	0	0	4	1
Ander.-Dar.	0	0	3	2
Vote Totals:	0	0	27	8

This distribution selection method selected the correct distribution in every case but the Exponential. This is due to the fact that the moment curves for the Weibull distribution intersect the point for the Exponential distribution. The Exponential distribution is actually a sub-set with specific parameters of the more general Weibull distribution. Therefore, Exponential samples scattered about the Exponential point have a tendency to be closer to the Weibull curve than the Exponential point. The selection method correctly selected the Normal distribution by a small margin over the Weibull distribution. Here, the goodness of fit statistics provided more accurate votes than the moment plots.

d. Varying the Sample Size

The test just described was based on drawing sample sizes of 1000 from a known distribution. The actual flight loads and fatigue life data to be analyzed with the method have sample sizes around 20. To determine how accurate this distribution selection method is with smaller sample sizes, the entire test was performed two more times. First, random samples of 100 were drawn from known distributions, analyzed with AGSS, the moments were plotted, goodness of fit statistics calculated, and the votes tallied. Then random samples of 20 were drawn, and the process was repeated. Figures 2.9a, b, c, and d show EDF/CDF plots and probability plots for Weibull samples fit to a Weibull distribution. Comparing these four Figures to Figures 2.5e and 2.5f gives a preliminary indication of how sample size affects the goodness of fit.

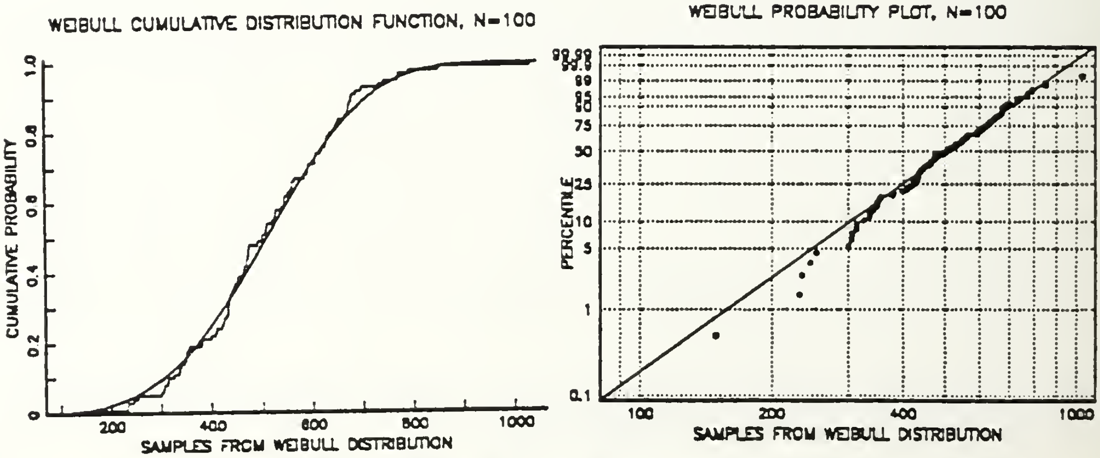


Figure 2.9a: EDF/CDF   Figure 2.9b: Probability
Random Weibull Sample of 100 fit to Weibull Distribution

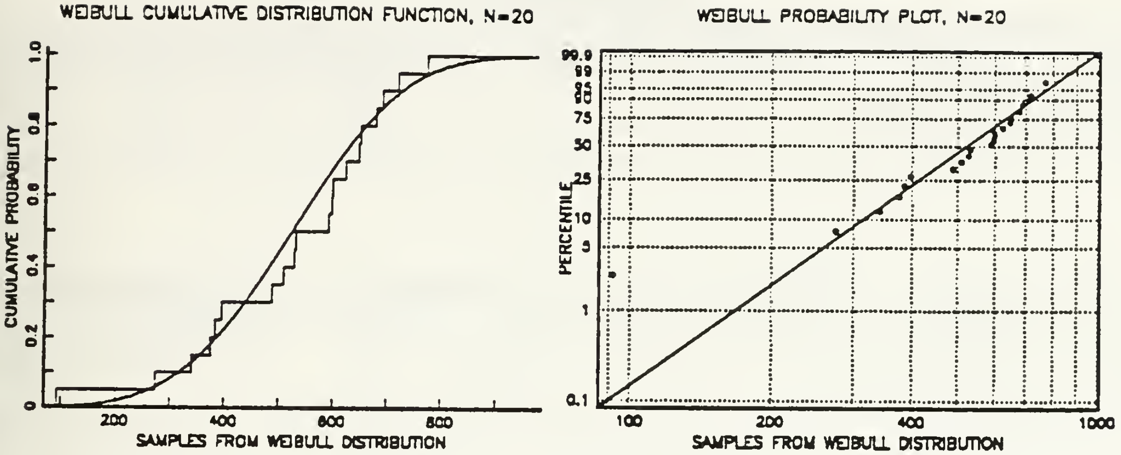


Figure 2.9c: EDF/CDF   Figure 2.9d: Probability
Random Weibull Sample of 20 fit to Weibull Distribution

Only the total vote count tables for each distribution are shown below. Tables 2.25 through 2.28 are for sample sizes of 100, Tables 2.29 trough 2.32 are for sample sizes of 20.

TABLE 2.25: SAMPLE SIZE OF 100 FROM NORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	11	2	7	0
Goodness of fit	7	3	6	0
Vote Totals:	18	5	13	0

TABLE 2.26: SAMPLE SIZE OF 100 FROM LOGNORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	5	4	6	0
Goodness of fit	0	16	4	0
Vote Totals:	5	20	10	0

TABLE 2.27: SAMPLE SIZE OF 100 FROM WEIBULL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	4	1	10	0
Goodness of fit	9	0	11	0
Vote Totals:	13	1	21	0

TABLE 2.28: SAMPLE SIZE OF 100 FROM EXPONENTIAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	0	14	7
Goodness of fit	0	0	16	4
Vote Totals:	0	0	30	11

TABLE 2.29: SAMPLE SIZE OF 20 FROM NORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	5	2	10	0
Goodness of fit	5	11	4	0
Vote Totals:	10	13	14	0

TABLE 2.30: SAMPLE SIZE OF 20 FROM LOGNORMAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	3	6	8	0
Goodness of fit	2	11	4	0
Vote Totals:	5	17	12	0

TABLE 2.31: SAMPLE SIZE OF 20 FROM WEIBULL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	5	3	13	0
Goodness of fit	5	3	9	0
Vote Totals:	10	6	22	0

TABLE 2.32: SAMPLE SIZE OF 20 FROM EXPONENTIAL DISTRIBUTION

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	0	15	2
Goodness of fit	0	3	7	5
Vote Totals:	0	3	22	7

The vote count method was just as effective with sample sizes of 100 as with 1000. Lognormal and Weibull distributions were correctly selected by a wide margin. Again, the Normal distribution was correctly selected by a small margin over the Weibull distribution with the moment plots indicating with greater accuracy, while Lognormal was also correctly selected, the goodness of fit tests were more accurate. Weibull was correctly selected with the moment plots indicating most accurately, but both methods missed on the Exponential. With sample sizes of 20 the method loses considerable accuracy. For the known Normal samples, moment plots indicated Weibull, goodness of fit statistics indicated Lognormal. However, the total votes for Normal was close to the totals for Lognormal and Weibull. Lognormal was correctly selected by a 3:2 margin, with the goodness of fit statistics providing the most accurate votes. Weibull was correctly selected by both components of the method, with a 2:1 margin. Based on the three test results of varying sample sizes, sometimes the moment plot is better, sometimes the goodness of fit tests are better. Combining the two, the distribution selection method is able to correctly determine whether a sample is from a Weibull or Lognormal distribution in most cases. When a sample has a coefficient of variation between 0.2 and 0.5, the moment plots method cannot differentiate between Normal and Weibull. The reason for this is that intersections occur for the Weibull and Normal distribution moment functions within this range (see Figure 2.4 and Figure 2.6). It would be reasonable to assume that if two distributions receive similar vote counts for a sample being analyzed, either distribution could be used to model the sample's population. Another alternative in case of a tie would be to tally the votes again, allowing each moment point and goodness of fit statistic to vote for one of the tied distributions only, after eliminating the others.

III. APPLICATIONS OF THE MOMENT METHOD

A. HELICOPTER FLIGHT LOADS DATA

The first application of the moment/goodness of fit vote count method was to flight loads measured on the main rotor forward longitudinal stationary star (MRFLSS) of the SH-60 Sikorsky helicopter. This is a fatigue critical component. Data was provided from two separate substantiation flights flown by United Technologies Corporation, Sikorsky Aircraft Division. Load measurements were recorded for a variety of maneuvers at a variety of airspeeds, collective settings, and "g" loadings. The maneuver that caused the most fatigue damaging loads was the symmetric pull-out. Table 3.1 lists the two flights at two different gross weights, and the runs made during each flight. Essential parameters of each run are also listed. Parameter abbreviations are: SP is symmetric pullout, 155K or 124K is the airspeed in knots, FC is fixed collective, TC is top collective, -25% is the collective set at 75% of TC, and G's are the accelerations the maneuver produced in terms of the acceleration of gravity.

TABLE 3.1: FLIGHT/RUN PARAMETERS

Flight 37: 16,500 gross wt.		Flight 43: 20,800 gross wt.	
RUN	**MANEUVER**	**RUN**	**MANEUVER**
14	SYM PULL (SP), 124K FIX COLL (FC), 2.6G	69	SP, 124K, 100% TORQUE (100%), 1.5G
15	SP, 124K, FC, 2.8G	70	SP, 124K, 100%, 1.75G
16	SP, 124K, FC, 2.8G	71	SP, 124K, 100%, 2.0G
17	SP, 124, FC, 3.0G	72	SP, 124K, 100%, 2.0G
19	SP, 124K -25%, 2.9G	73	SP, 124K, 100%, 2.1G
21	SP, 124K -25%, 2.8G	74	SP, 124K, 100%, 2.1G
22	SP, 124K, -25%, 3.0G	75	SP, 124K, 100%, 1.75G
23	SP, 124K, -25%, 2.9G	76	SP, 124K, 100%, 2.1G
24	SP, 124K, -25%, 2.9G	77	SP, 124K, 100%, 2.3G
25	SP, 124K, TOP COLL (TC), 2.8G	81	SP, 155K, 100%, 1.5G
28	SP, 155K, -25%, 2.5G	82	SP, 155K, 100%, 1.75G
29	SP, 155K, -25%, 3.4G	83	SP, 155K, 100%, 2.1G
30	SP, 155K, -25%, 3.1G	84	SP, 155K, 105%, 1.75G
31	SP, 155K, -25%, 3.1G	85	SP, 155K, 105%, 2.1G
		86	SP, 155K, 100%, 1.5G
		87	SP, 155K, 100%, 1.85G
		88	SP, 155K, 100%, 2.3G

1. Loads Data Processing

From a statistical standpoint, it would be ideal to have several runs made for each set of parameters; rather than a different set of parameters for each run. However, this is not deemed economically feasible with substantiation flights at the present time; therefore, the flight loads data had to be pooled prior to any statistical analysis.

a. Sorting and Pooling Data

Load versus time plots were made of each run. Flight 43, which had the higher gross weight, produced higher loads, even though the "g" loading was slightly less than flight 37. Within the two flights, the runs at 155 kts. produced slightly higher loads than those at 124 kts. Figures 3.1 and 3.2 are examples of the load versus time plots. Figure 3.1 is at 124 kts.: Figure 3.2 is at 155 kts.

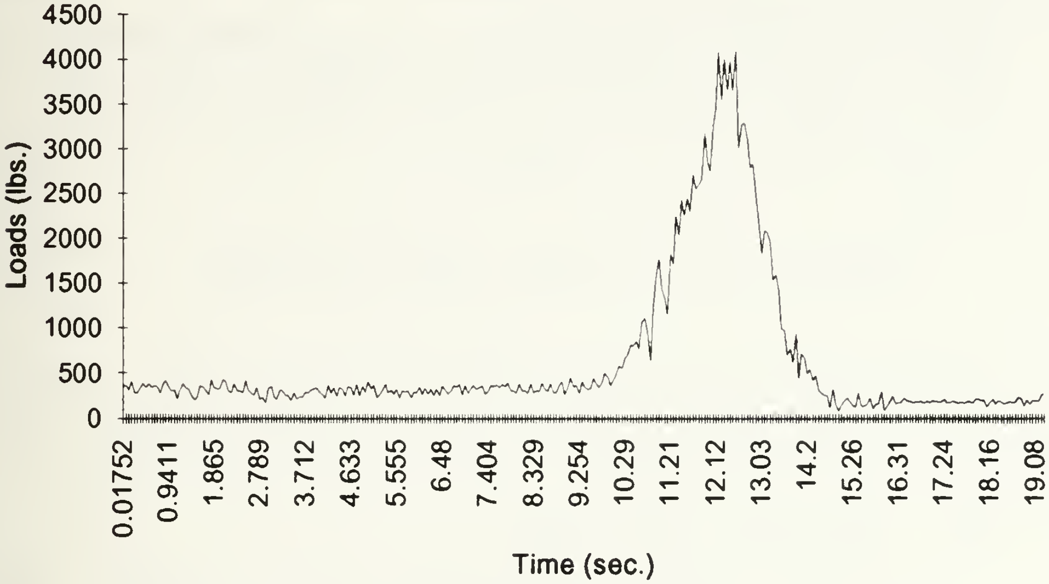


Figure 3.1: 124 kts. Load vs. Time

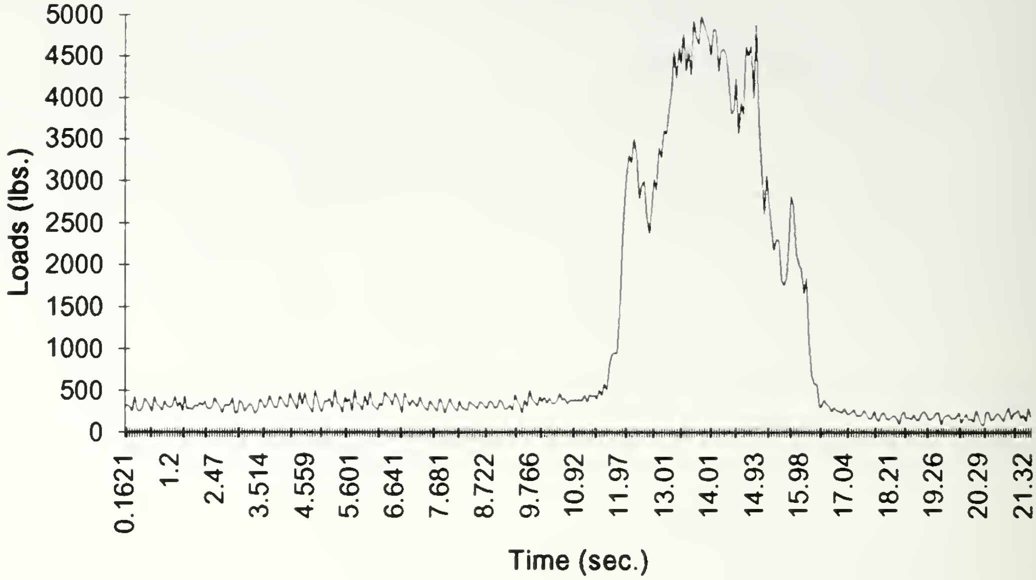


Figure 3.2: 155 kts. Load vs. Time

Initially four pools were established, one for each flight speed within the two flights.

b. Selecting Peak Loads

Fatigue life calculations only consider the maximum and minimum loads in a given load cycle. Because these loads were sampled every 0.17 seconds, many of the loads recorded were intermediate loads en route to a maximum or minimum. The loads of each run were processed through a computer algorithm to pick out only the maximum loads, or "peaks", which are the loads of principal interest. If needed, the same analysis could be carried out to find the minimum loads in each excursion, or what are called the "valleys".

c. Selecting Cut-off Loads

In addition to helping pool the data, the load versus time plots indicated that the recording instruments were turned on just before the maneuver was initiated, and turned off just after the maneuver was completed. Loads encountered in straight and level flight are not fatigue damaging, and are not necessarily from the same population as the fatigue damaging loads produced during the maneuver. A minimum cut-off load was determined for each pool of data. Loads less than the cut-off load were not considered in the analysis. The cut-off load was determined by plotting the empirical cumulative distribution function for the loads of each run. Each of these plots produced a significant cusp, which indicated the possibility of two populations. One for the straight and level flight loads and one for the maneuver loads. The peak of the cusp is taken as the cut-off load. [Ref. 8] Figures 3.3 and 3.4 are typical empirical cumulative distribution function plots. Figure 3.3 is a plot of all the recorded loads. Figure 3.4 is a plot of only the maximum loads.

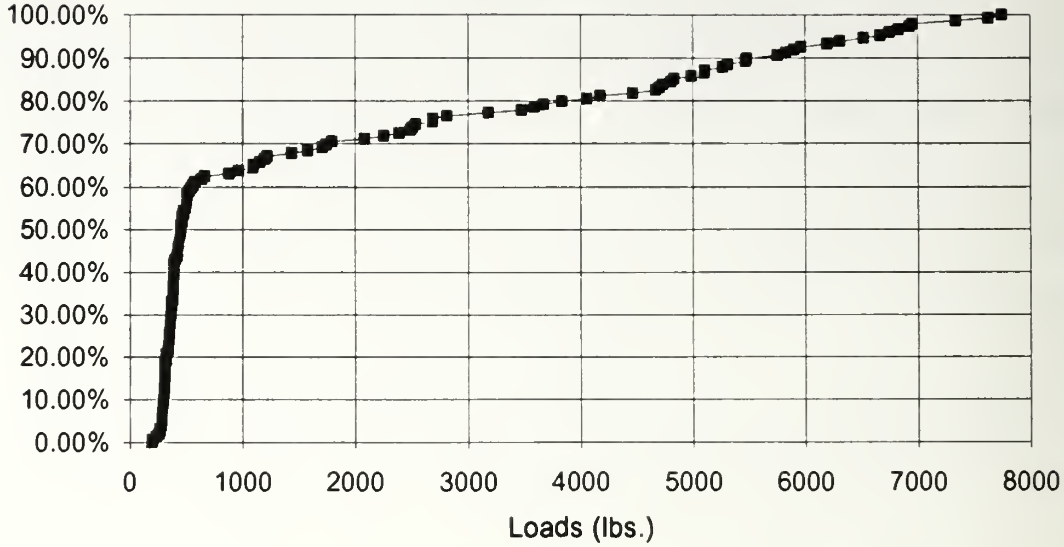


Figure 3.3: ECDF for all Loads of Flight 43 Run 88.

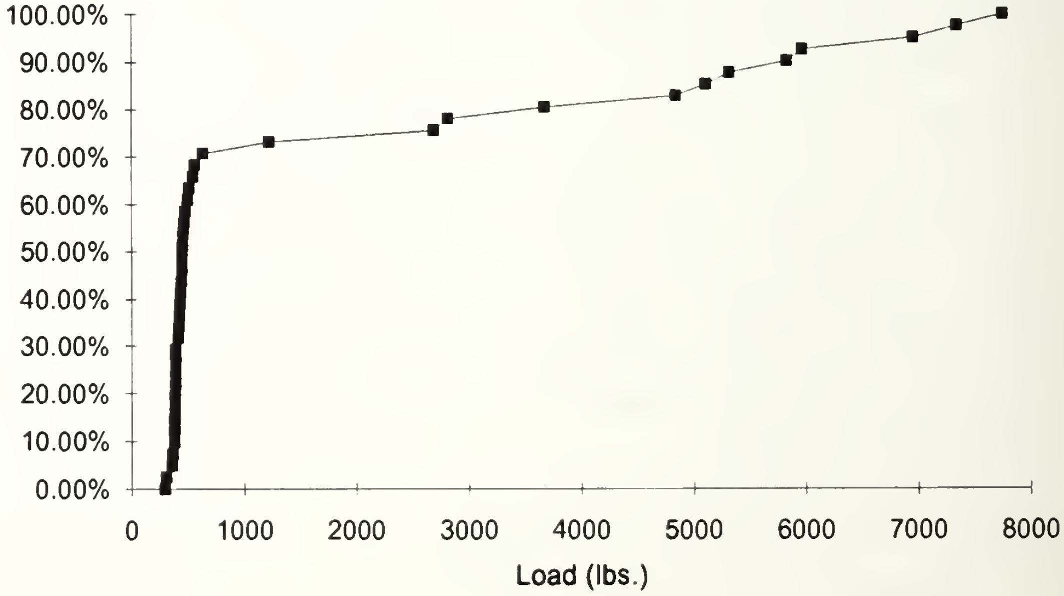


Figure 3.4: ECDF for Peak Loads of Flight 43 Run 88

The ECDF's for the flight 43 runs indicated a cut-off load of 750 lbs. for both flight speeds. The ECDF's for the flight 37 runs indicated cut-off loads of 500 lbs. for runs flown at 124 kts. and 1400 lbs. for runs flown at 155 kts.

2. Selecting the Best Distribution

Once the peak loads above the cut-off were selected from each run, all the runs were analyzed with AGSS to obtain the moment values and goodness of fit statistics.

a. Moment Plots

Three moment plots were made for each of the two flights. Figures 3.5, 3.6, and 3.7 are for Flight 37, and Figures 3.8, 3.9, and 3.10 are for Flight 43. Each data point on the plots represents a run from that particular flight.

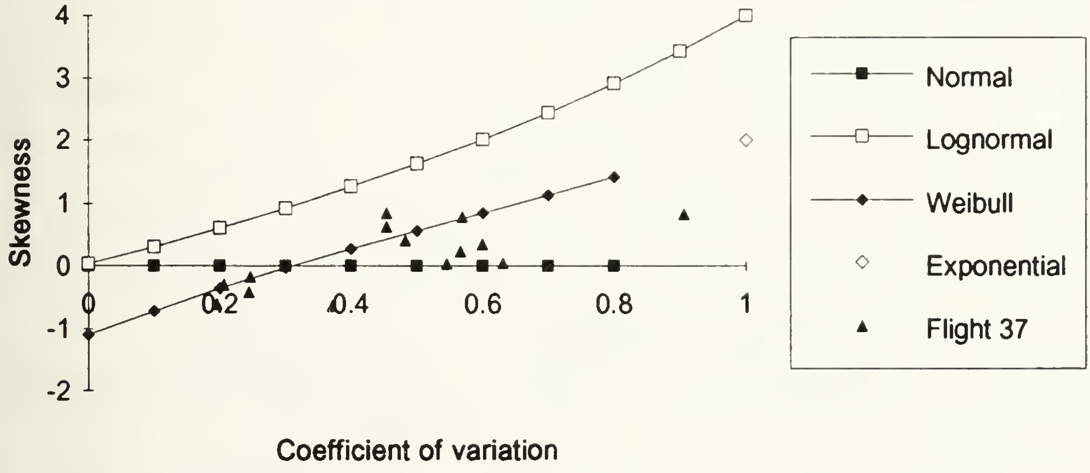


Figure 3.5: Flight 37 Coefficient of Variation vs. Skewness

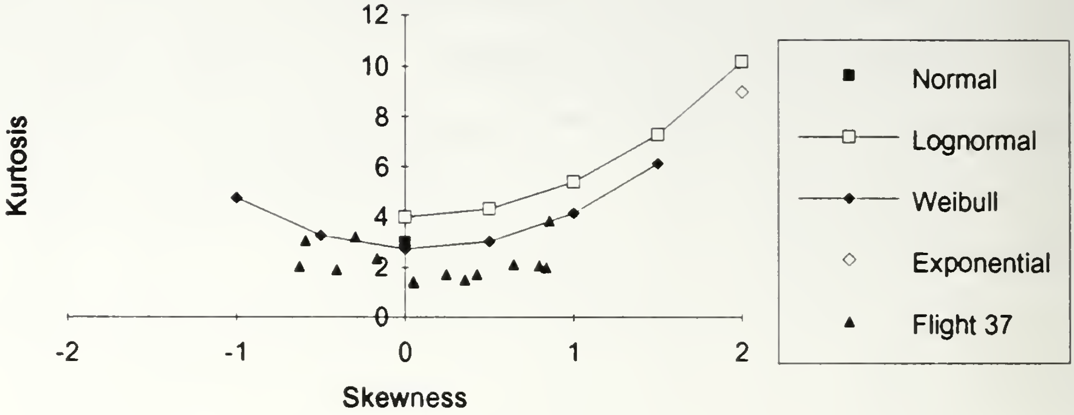


Figure 3.6: Flight 37 Kurtosis vs. Skewness

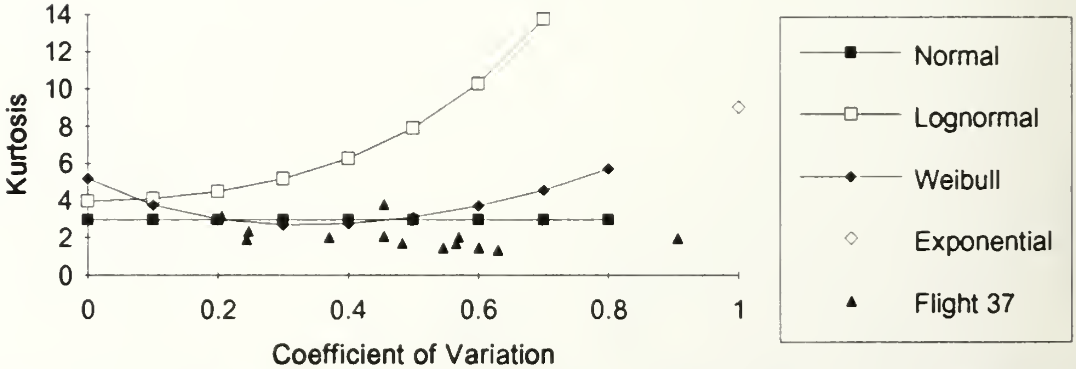


Figure 3.7: Flight 37 Kurtosis vs. Coefficient of Variation

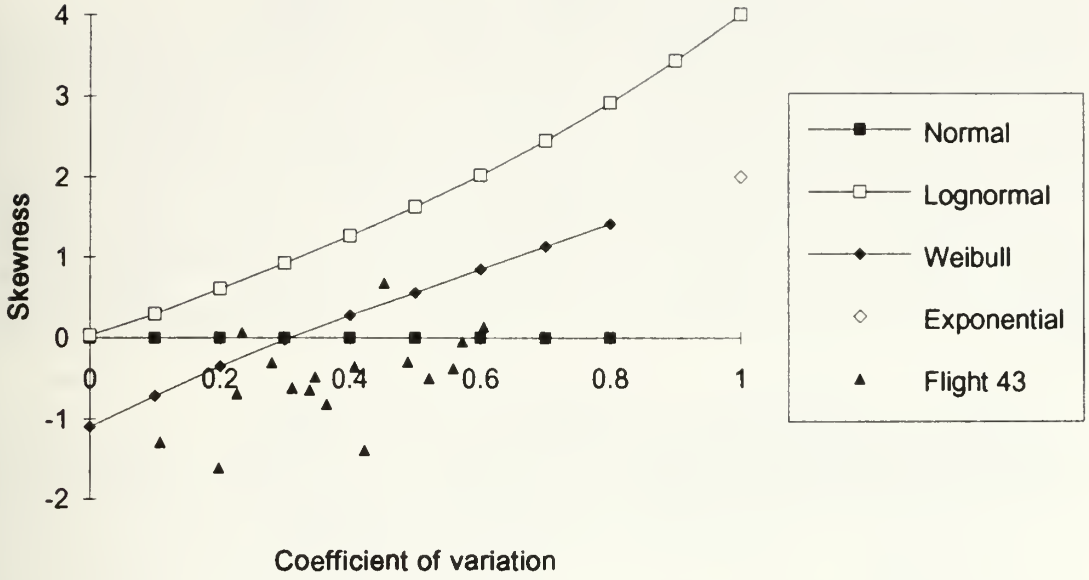


Figure 3.8: Flight 43 Skewness vs. Coefficient of Variation

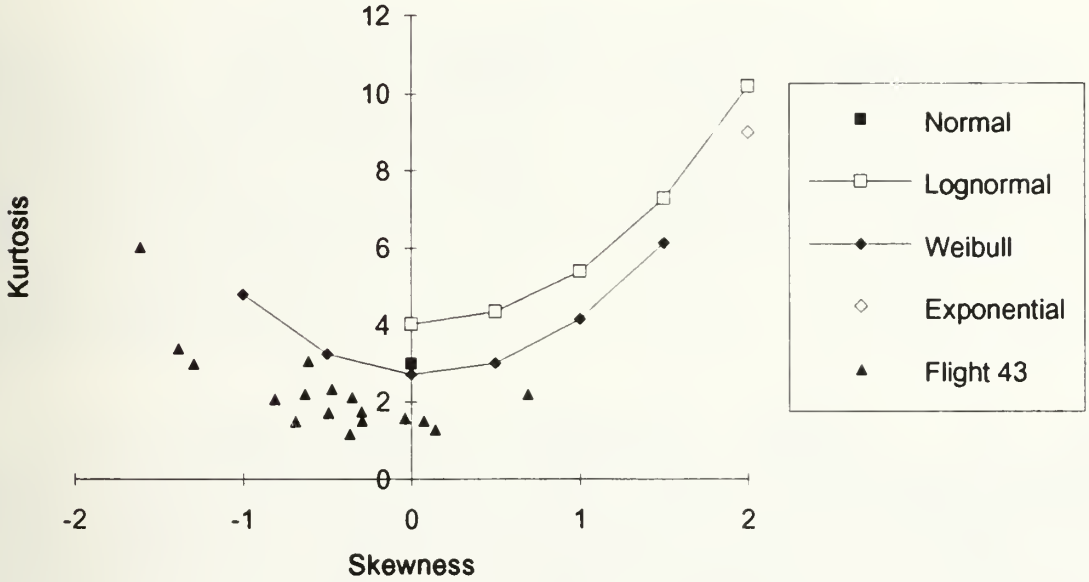


Figure 3.9: Flight 43 Kurtosis vs. Skewness

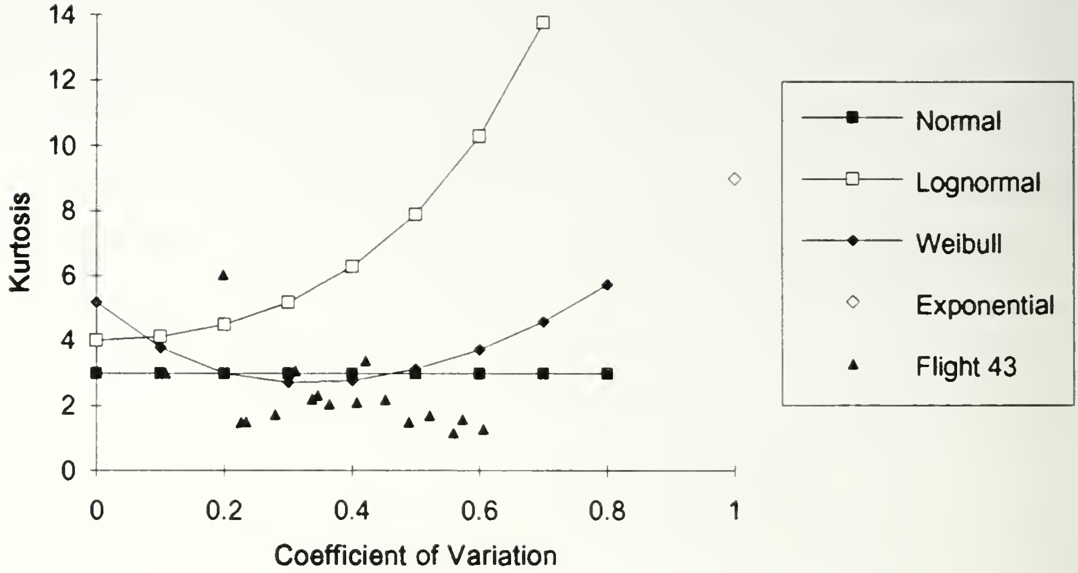


Figure 3.10: Flight 43 Kurtosis vs. Coefficient of Variation

b. Goodness of Fit Results

Tables 3.2 through 3.15 contain the goodness of fit statistics for flight 37. Tables 3.16 through 3.32 are for flight 43. Just as each run was represented by a point on the moment plots, each goodness of fit table represents the indicated run.

Table 3.2: Flight 37, Run 14

Test \ Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	1.0311	0.86079	1.1023	4.0513
Significance	0.30991	0.35352	0.29376	0.044135
Kolm-Smirn	0.16464	0.21922	0.17009	0.31026
Significance	0.81084	0.4668	0.77833	0.11137
Cramer-V M	0.086368	0.16891	0.094401	0.54041
Significance	>0.15	>0.15	>0.15	<0.05
Ander-Darl	0.58934	1.0447	0.70305	2.775
Significance	>0.15	>0.15	>0.15	<0.05

TABLE 3.3: FLIGHT 37, RUN 15

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4.2821	2.8581	2.6263	4.4395
Significance	0.23257	0.23953	0.26897	0.21775
Kolm-Smirn	0.13109	0.12072	0.13138	0.22027
Significance	0.78353	0.85947	0.78121	0.17667
Cramer-V M	0.077271	0.11034	0.067144	0.33098
Significance	>0.15	>0.15	>0.15	<0.15
Ander-Darl	0.6294	0.71922	0.53855	1.9935
Significance	>0.15	>0.15	>0.15	<0.1

TABLE 3.4: FLIGHT 37, RUN 16

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4.1774	2.1253	1.9751	1.1703
Significance	0.040967	0.14489	0.15991	0.27933
Kolm-Smirn	0.15847	0.1805	0.16679	0.24436
Significance	0.78675	0.63688	0.73159	0.26203
Cramer-V M	0.097685	0.11795	0.097936	0.26596
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.65443	0.75211	0.66145	1.591
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.5: FLIGHT 37, RUN 17

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	5.1403	3.4622	3.5367	2.7349
Significance	0.076518	0.062793	0.060023	0.25475
Kolm-Smirn	0.17202	0.20693	0.17289	0.19776
Significance	0.62749	0.39001	0.62112	0.44727
Cramer-V M	0.14312	0.16876	0.1522	0.19064
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.91033	1.0027	0.94135	1.2287
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.6: FLIGHT 37, RUN 19

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.4927	0.64682	1.008	10.414
Significance	0.2218	0.42125	0.31538	0.0054792
Kolm-Smirn	0.18772	0.13814	0.16516	0.35386
Significance	0.51475	0.86144	0.67786	0.017161
Cramer-V M	0.13295	0.055689	0.097098	0.56387
Significance	>0.15	>0.15	>0.15	<0.05
Ander-Darl	0.79197	0.38517	0.58641	2.9177
Significance	>0.15	>0.15	>0.15	<0.05

TABLE 3.7: FLIGHT 37, RUN 21

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				5.6639
Significance				0.017319
Kolm-Smirn	0.2569	0.20094	0.23811	0.34982
Significance	0.35751	0.67018	0.45247	0.083028
Cramer-V M	0.10595	0.081631	0.08887	0.33535
Significance	>0.15	>0.15	>0.15	<0.15
Ander-Darl	0.60164	0.45498	0.49567	1.7993
Significance	>0.15	>0.15	>0.15	<0.15

TABLE 3.8: FLIGHT 37, RUN 22

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				2.349
Significance				0.12537
Kolm-Smirn	0.23947	0.21857	0.20339	0.26797
Significance	0.20155	0.29493	0.37962	0.11311
Cramer-V M	0.2258	0.12369	0.16535	0.27805
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	1.219	0.8353	0.94491	1.7075
Significance	>0.15	>0.15	>0.15	<0.15

TABLE 3.9: FLIGHT 37, RUN 23

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				12.017
Significance				0.00052727
Kolm-Smirn	0.15202	0.15757	0.13444	0.31143
Significance	0.85343	0.82181	0.93457	0.089768
Cramer-V M	0.0468	0.062458	0.042465	0.49174
Significance	>0.15	>0.15	>0.15	<0.1
Ander-Darl	0.3444	0.35505	0.28503	2.5083
Significance	>0.15	>0.15	>0.15	<0.05

TABLE 3.10: FLIGHT 37, RUN 24

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.9906		2.1593	2.0009
Significance	0.08375		0.14171	0.1572
Kolm-Smirn	0.25282	0.17622	0.21791	0.33561
Significance	0.29301	0.74016	0.47452	0.068152
Cramer-V M	0.19108	0.07813	0.12792	0.32381
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	1.0945	0.52289	0.75556	1.7684
Significance	>0.15	>0.15	>0.15	<0.15

TABLE 3.11: FLIGHT 37, RUN 25

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	17.463	12.977	11.923	8.783
Significance	0.00056746	0.0015209	0.0025762	0.066747
Kolm-Smirn	0.27965	0.15802	0.19394	0.20625
Significance	0.006133	0.31394	0.12362	0.085889
Cramer-V M	0.65966	0.2949	0.37128	0.2822
Significance	<0.025	>0.15	<0.1	>0.15
Ander-Darl	3.7005	2.0119	2.2815	1.9219
Significance	<0.025	<0.1	<0.1	<0.15

TABLE 3.12: FLIGHT 37, RUN 28

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.6221	2.6723	2.1401	65.469
Significance	0.20279	0.10211	0.14349	6.0646E-15
Kolm-Smirn	0.095733	0.13483	0.10274	0.46219
Significance	0.98432	0.79721	0.96842	0.00010801
Cramer-V M	0.034765	0.077259	0.038166	1.4453
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.27671	0.56491	0.29527	6.8511
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.13: FLIGHT 37, RUN 29

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4.9526	5.6867	4.9137	37.814
Significance	0.08405	0.058235	0.0857	6.1479E-9
Kolm-Smirn	0.11839	0.15903	0.11882	0.4307
Significance	0.77771	0.41313	0.7739	0.000020235
Cramer-V M	0.058111	0.086874	0.065165	1.6955
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.38266	0.54826	0.41088	8.1674
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.14: FLIGHT 37, RUN 30

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.5032	2.473	1.1959	51.488
Significance	0.22018	0.2904	0.27415	6.5988E-12
Kolm-Smirn	0.21677	0.25182	0.18506	0.43686
Significance	0.25248	0.12279	0.4384	0.00045089
Cramer-V M	0.10192	0.16881	0.070557	1.417
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.50851	0.88421	0.35795	6.7205
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.15: FLIGHT 37, RUN 31

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	6.5109	7.2702	6.6146	31.556
Significance	0.038556	0.02638	0.036616	1.4052E-7
Kolm-Smirn	0.22611	0.22167	0.22609	0.41391
Significance	0.15511	0.1713	0.1552	0.00038099
Cramer-V M	0.16211	0.15717	0.17622	1.3739
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.9437	0.98698	1.0007	6.6571
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.16: FLIGHT 43, RUN 69

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.39305	0.40496	0.34178	0.55623
Significance	0.42254	0.38516	0.60325	0.090641
Cramer-V M	0.13641	0.14756	0.098794	0.42054
Significance	>0.15	>0.15	>0.15	<0.15
Ander-Darl	0.75036	0.8066	0.63501	1.9441
Significance	>0.15	>0.15	>0.15	<0.1

TABLE 3.17: FLIGHT 43, RUN 70

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.20637	0.17578	0.21794	0.49659
Significance	0.88508	0.96573	0.84181	0.038679
Cramer-V M	0.052787	0.049248	0.052623	0.47269
Significance	>0.15	>0.15	>0.15	<0.1
Ander-Darl	0.32153	0.30119	0.323	2.2578
Significance	>0.15	>0.15	>0.15	<0.1

TABLE 3.18: FLIGHT 43, RUN 71

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.16832	0.20853	0.15926	0.44298
Significance	0.98881	0.92108	0.99429	0.12817
Cramer-V M	0.035676	0.050996	0.032421	0.36011
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.23232	0.31736	0.22218	1.7605
Significance	>0.15	>0.15	>0.15	<0.15

TABLE 3.19: FLIGHT 43, RUN 72

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.28054	0.30179	0.29341	0.34571
Significance	0.73248	0.6453	0.67992	0.47017
Cramer-V M	0.099269	0.11644	0.10509	0.25719
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.5858	0.69256	0.66857	1.3046
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.20: FLIGHT 43, RUN 73

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.18184	0.27709	0.21084	0.29977
Significance	0.92724	0.49422	0.81853	0.3937
Cramer-V M	0.072424	0.13919	0.099237	0.19355
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.46144	1.81086	0.65584	1.0421
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.21: FLIGHT 43, RUN 74

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.34549	0.36696	0.35708	0.33555
Significance	0.58938	0.51109	0.54665	0.62663
Cramer-V M	0.1193	0.1332	0.13177	0.13708
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.70424	0.77962	0.79783	0.76411
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.22: FLIGHT 43, RUN 75

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.27956	0.3189	0.29116	0.3067
Significance	0.82939	0.68937	0.79039	0.73472
Cramer-V M	0.062058	0.073795	0.067721	0.13893
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.34847	0.4262	0.39047	0.75233
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.23: FLIGHT 43, RUN 76

Test \ Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square				
Significance				
Kolm-Smirn	0.26591	0.22184	0.24307	0.25398
Significance	0.62363	0.82599	0.73192	0.68047
Cramer-V M	0.099124	0.088163	0.094024	0.12594
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.57631	0.50416	0.53411	0.76299
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.24: FLIGHT 43, RUN 77

Test \ Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square				
Significance				
Kolm-Smirn	0.19214	0.27876	0.23158	0.27585
Significance	0.85407	0.41875	0.65698	0.43207
Cramer-V M	0.067287	0.10489	0.08097	1.1563
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.41626	0.58914	0.48493	0.9025
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.25: FLIGHT 43, RUN 81

Test \ Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	2.881	1.7643	3.0368	10.49
Significance	0.089624	0.41388	0.081397	0.0052747
Kolm-Smirn	0.15405	0.17616	0.1558	0.37964
Significance	0.67343	0.50203	0.65958	0.0035231
Cramer-V M	0.12011	0.15818	0.12284	0.88056
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.69319	1.1092	0.74368	4.374
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.26: FLIGHT 43, RUN 82

Test \ Fit	**Normal**	**Lognormal**	**Weibull**	**Exponential**
Chi-Square	2.4177	8.319	2.0645	36.881
Significance	0.11997	0.015614	0.15077	9.806E-9
Kolm-Smirn	0.10457	0.1879	0.099637	0.39428
Significance	0.95555	0.36505	0.97104	0.001149
Cramer-V M	0.051201	0.21903	0.045627	1.0905
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.3638	1.3328	0.37743	5.3304
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.27: FLIGHT 43, RUN 83

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.7166	1.1838	1.974	14.553
Significance	0.25709	0.27659	0.3727	0.00069182
Kolm-Smirn	0.20433	0.13358	0.18294	0.38072
Significance	0.37397	0.8677	0.51494	0.0060678
Cramer-V M	0.15565	0.073333	0.11576	0.60754
Significance	>0.15	>0.15	>0.15	<0.05
Ander-Darl	0.88484	0.49288	0.67285	3.1335
Significance	>0.15	>0.15	>0.15	<0.025

TABLE 3.28: FLIGHT 43, RUN 84

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.39403	0.40034	0.39173	0.52289
Significance	0.74018	0.72218	0.74666	0.38495
Cramer-V M	0.089017	0.092442	0.090408	0.20654
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.51373	0.53189	0.55786	0.97866
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.29: FLIGHT 43, RUN 85

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.25698	0.38104	0.27928	0.43714
Significance	0.82303	0.34834	0.73753	0.20169
Cramer-V M	0.11961	0.20696	0.13945	0.2601
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.74355	1.1421	0.98897	1.2522
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.30: FLIGHT 43, RUN 86

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				6.0039
Significance				0.014274
Kolm-Smirn	0.20364	0.27296	0.1564	0.5081
Significance	0.60712	0.24786	0.88324	0.001451
Cramer-V M	0.098102	0.22316	0.049068	0.94515
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.71749	1.4096	0.45213	4.4551
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.31: FLIGHT 43, RUN 87

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4.4538	3.4026	4.8436	19.373
Significance	0.034825	0.065098	0.027749	0.000062117
Kolm-Smirn	0.20776	0.24073	0.2065	0.37559
Significance	0.38507	0.22085	0.39257	0.0093973
Cramer-V M	0.15245	0.20253	0.15922	0.78203
Significance	>0.15	>0.15	>0.15	<0.025
Ander-Darl	0.83895	1.1936	0.90796	3.9292
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.32: FLIGHT 43, RUN 88

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				
Significance				
Kolm-Smirn	0.14148	0.23012	0.15441	0.33498
Significance	0.96992	0.54874	0.93718	0.13531
Cramer-V M	0.037687	0.096706	0.044418	0.38201
Significance	>0.15	>0.15	>0.15	<0.15
Ander-Darl	0.24575	0.60483	0.30691	1.9562
Significance	>0.15	>0.15	>0.15	<0.1

c. *Vote Tally*

Tables 3.33 and 3.34 contain the vote counts for flights 37 and 43 respectively.

TABLE 3.33: FLIGHT 37 VOTE COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	13	0	30	0
Goodness of fit	18	16	12	6
Vote Totals:	31	16	42	6

TABLE 3.34: FLIGHT 43 VOTE COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	15	0	30	0
Goodness of fit	28	12	13	1
Vote Totals:	43	12	33	1

The goodness of fit statistics chose a Normal distribution for both flights by a small margin, while the moment plotting method chose Weibull for both flights by a large margin. The total votes indicate that the flight 37 load samples are from a Weibull distribution, while the load samples from flight 47 are from a Normal distribution. The methods having eliminated all but Normal and Weibull, the votes were tallied again, allowing the points and goodness of fit statistics to vote solely for Normal or Weibull distributions. The following tables show the resulting vote totals.

TABLE 3.35: FLIGHT 37 VOTE COUNTS

TEST\DISTRIBUTION	**Normal**	**Weibull**
Moment Plots	13	30
Goodness of fit	21	32
Vote Totals:	34	62

TABLE 3.36: FLIGHT 43 VOTE COUNTS

TEST\DISTRIBUTION	**Normal**	**Weibull**
Moment Plots	15	30
Goodness of fit	33	22
Vote Totals:	48	52

These tables strengthen the case for a Weibull distribution for flight 37, and raise doubts about flight 43 having a Normal Distribution. It seems unlikely that simply changing the gross weight of the aircraft would change the load distribution. If curve fits were made of the data, it is clear that plots on Figure 3.5 would indicate Weibull-like behavior with a positive slope. On Figure 3.6 a curve fit indicates Weibull-like rather than Lognormal; thus the moment method strongly indicates Weibull for flight 37. Similar arguments point to Weibull for flight 43 also; thus a curve fit approach would confirm the moment methods vote results.

C. A-6 COUNTING ACCELEROMETER DATA

1. Explanation of Data

The third application of the moment plot/goodness of fit vote counting method was to counting accelerometer measurements from a pool of 103, A-6 aircraft. The accelerometers count the number of exceedences of 4, 5, 6, and 7 "g" accelerations. They are used to track the fatigue lives of each aircraft by recording the fatigue damaging loads each aircraft experiences. The recorded loads are then converted, through methods of fatigue analysis, into a percentage of fatigue life expended (FLE). Once an aircraft reaches an FLE of 0.67, it has consumed 67% of its fatigue life and is restricted to lower flight accelerations. [Ref. 9] The pool of 103 were extracted from a population of 351 A-6 aircraft in the Navy's inventory that possessed usable data. The 103 selected were all the flight unrestricted aircraft in the population. The values of the random variable were the number of exceedences of a given g load per aircraft per 1000 flight hours. The data were converted from exceedence data to occurrences, where 7 g's do not also count as occurrences of 4, 5, and 6 g's.

2. Moment Plots

AGSS was used to compute sample values of the coefficient of variation, skewness, and kurtosis for each g loading. This application of the moment plot method used a different region of the template. The number of g occurrences varied greatly between aircraft, particularly at the 7 g level, where only a few aircraft had any occurrences at all. This produced large standard deviations. Sample coefficients of variation ranged from 0.6 to 2.47, compared to coefficients of variation for the helicopter flight loads which ranged from 0.1 to 0.9.

Figures 3.11 through 3.13 are the moment plots. Each plot has four points representing the four different g loads. Each point is calculated from the data of all 103 aircraft.

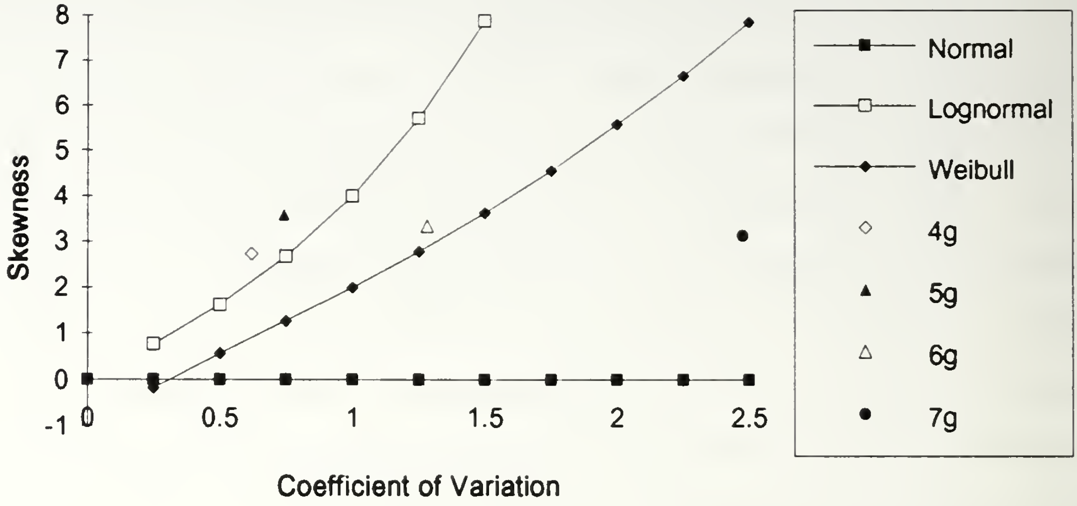


Figure 3.11: Skewness vs. Coefficient of Variation

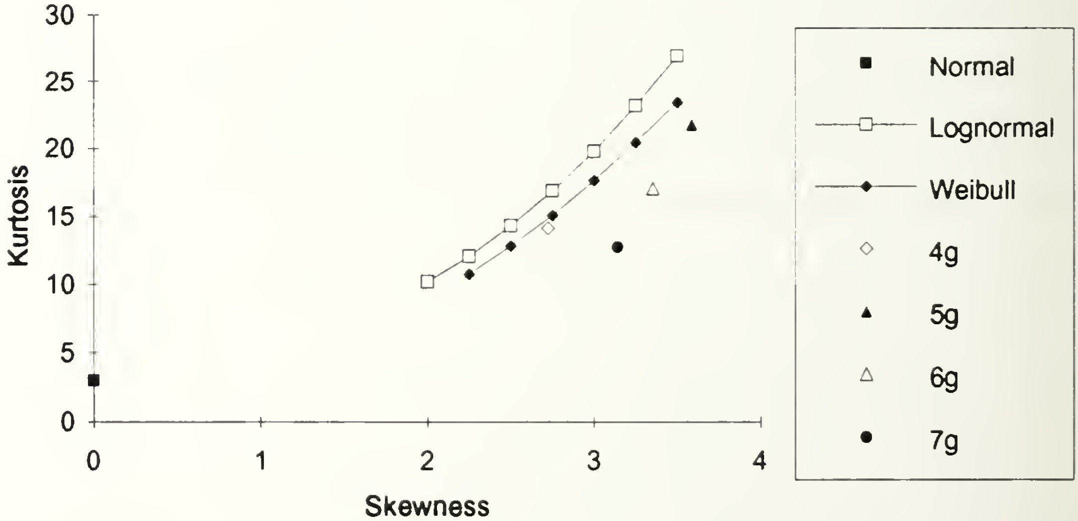


Figure 3.12: Kurtosis vs. Skewness

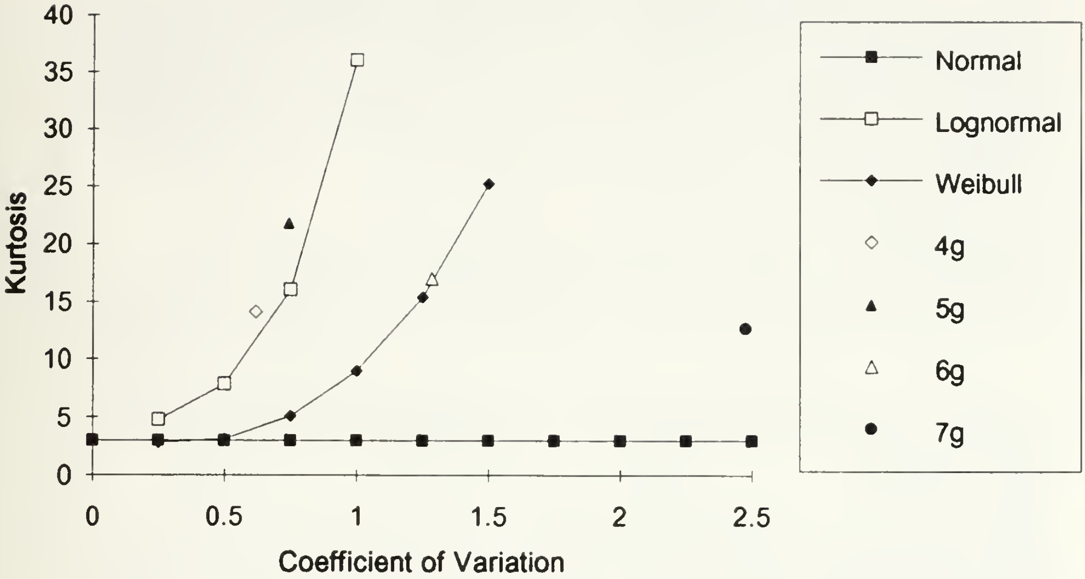


Figure 3.13: Kurtosis vs. Coefficient of Variation

3. Goodness of Fit Results

The goodness of fit statistics AGSS calculated are shown in tables 3.37 through 3.40. Each g loading has a set of values. This data was only fit to the three distributions indicated.

TABLE 3.37: 4G OCCURRENCES

Test \ Fit	**Normal**	**Lognormal**	**Weibull**
Chi-Square	11.441	2.8789	5.5232
Significance	0.0032783	0.23704	0.083194
Kolm-Smirn	0.15585	0.067953	0.10747
Significance	0.013431	0.72839	0.18511
Cramer-V M	0.69635	0.060641	0.33424
Significance	<0.025	>0.15	<0.15
Ander-Darl	4.3708	0.38485	2.4349
Significance	<0.01	>0.15	<0.1

TABLE 3.38: 5G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull
Chi-Square	12.764	5.401	8.5829
Significance	0.00035342	0.020126	0.0033935
Kolm-Smirn	0.17077	0.11513	0.11894
Significance	0.0049197	0.13035	0.10849
Cramer-V M	0.93956	0.26536	0.40923
Significance	<0.01	>0.15	<0.1
Ander-Darl	5.6594	1.7551	2.5187
Significance	<0.01	<0.15	<0.05

TABLE 3.39: 6G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull
Chi-Square	13.826	33.864	2.6483
Significance	0.00020048	2.1163E-7	0.26602
Kolm-Smirn	0.21682	0.28995	0.15505
Significance	0.0001245	6.0187E-8	0.014131
Cramer-V M	1.7849	2.555	0.45009
Significance	<0.01	<0.01	<0.1
Ander-Darl	9.732	14.603	3.5336
Significance	<0.01	<0.01	<0.025

TABLE 3.40: 7G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull
Chi-Square	39.264		
Significance	3.7018E-10		
Kolm-Smirn	0.34419	0.43237	0.41699
Significance	5.0384E-11	3.7696E-17	5.56E-16
Cramer-V M	4.6253	3.6886	3.4309
Significance	<0.01	<0.01	<0.01
Ander-Darl	22.925	19.768	17.989
Significance	<0.01	<0.01	<0.01

4. Vote Tally

Tables 3.41 through 3.44 contain the vote count results from the accelerometer data.

TABLE 3.41: 4G VOTES COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull
Moment Plots	0	2	1
Goodness of fit	0	4	0
Vote Totals:	0	6	1

TABLE 3.42: 5G VOTE COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull
Moment Plots	0	2	1
Goodness of fit	0	4	0
Vote Totals:	0	6	1

TABLE 3.43: 6G VOTE COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull
Moment Plots	0	0	3
Goodness of fit	0	0	4
Vote Totals:	0	0	7

TABLE 3.44: 7G VOTE COUNTS

TEST\DISTRIBUTION	Normal	Lognormal	Weibull
Moment Plots	2	0	2
Goodness of fit	1	0	2
Vote Totals:	3	0	4

The distribution selection method chose a Lognormal distribution for the 4g and 5 g occurrences, and a Weibull distribution for the 6 g and 7 g occurrences. From the earlier random number tests of the method, it was determined that the method was most accurate at selecting Lognormal and Weibull distributions. That does not mean that the method is biased toward those distributions, but rather, when the random sample was from a Lognormal distribution, it chose that distribution by a wide margin. The same was true for a random sample from a Weibull distribution.

D. AGARD A-7 COUNTING ACCELEROMETER DATA

1. Explanation of data

The following analysis was performed on data taken from AGARD Conference Proceedings 506 [Ref. 10]. It was chosen because it contained more data for the high g loads, than the A-6 application above. Data was available on 40 aircraft. It was in terms of exceedences and had been standardized to exceedences per 1000 hours. The moment plot/goodness of fit statistics vote count method was performed on the data twice. Once on the exceedence data, and again on the data after it had been converted to occurrences. The object was to see if the distributions were affected by the form of the counts, i.e., exceedences or occurrences.

2. Moment Plots

Figures 3.14 through 3.16 are the moment plots for the data in exceedence form. Figures 3.17 through 3.19 are the moment plots for the data in occurrence form.

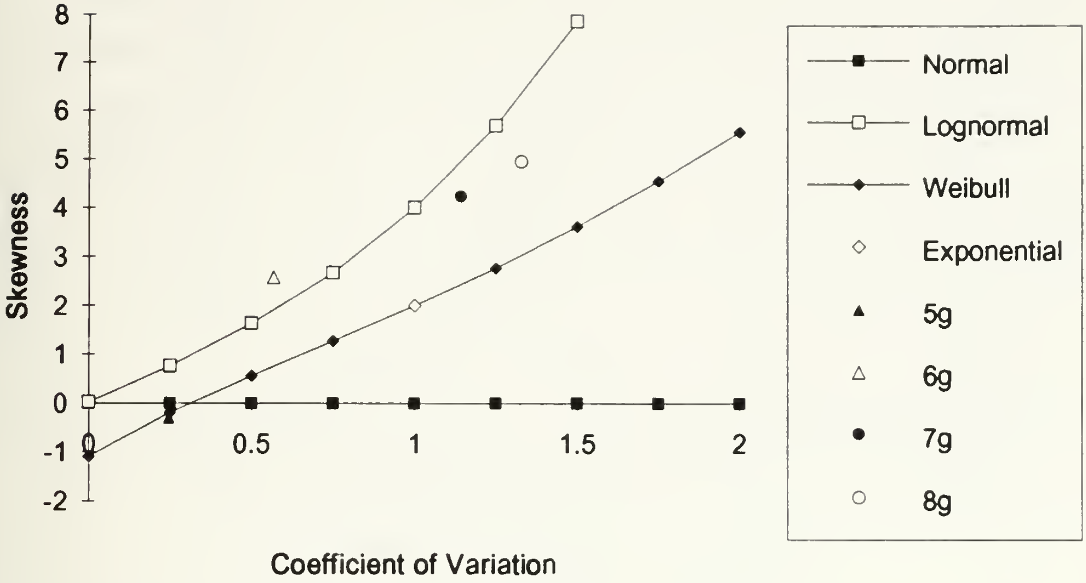


Figure 3.14: Skew. vs. Coef. of Var. for Exceedence Data

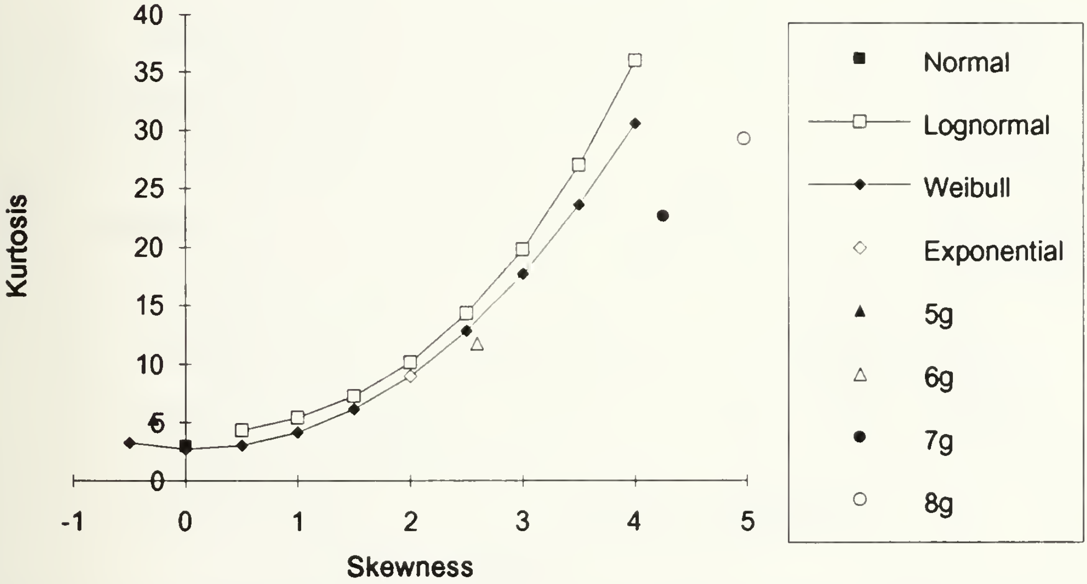


Figure 3.15: Kurt. vs. Skew. for Exceedence Data

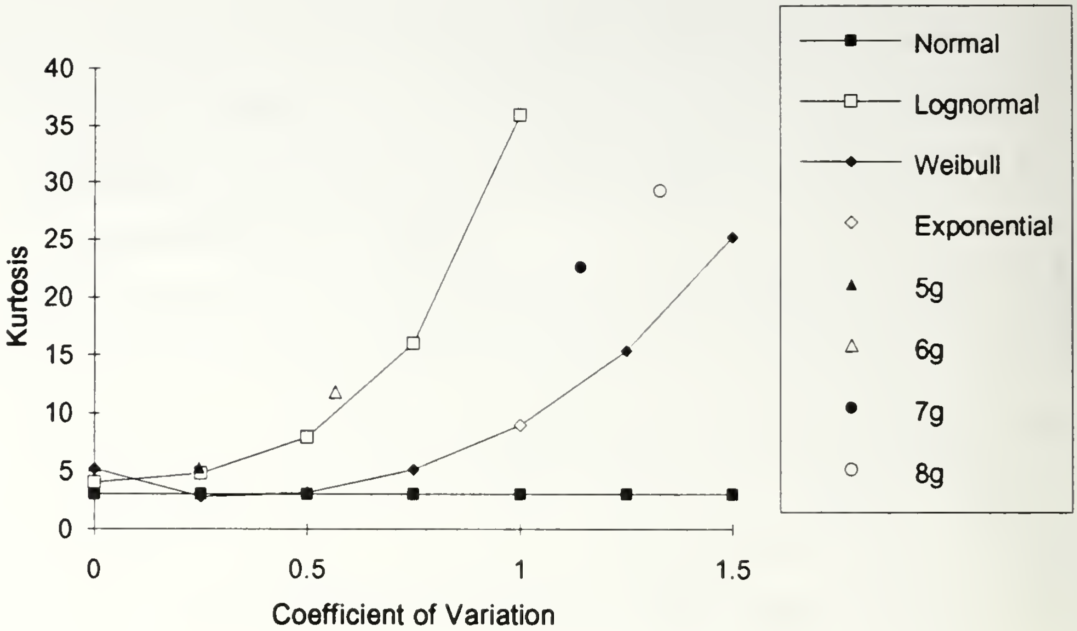


Figure 3.16: Kurt. vs. Coef. of Var. for Exceedence Data

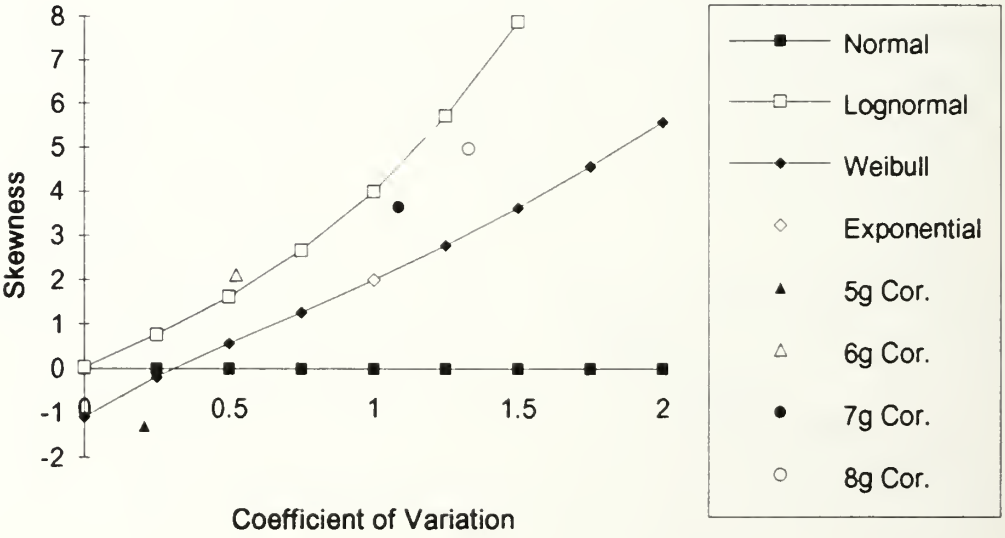


Figure 3.17: Skew. vs. Coef. of Var. for Occurrence Data

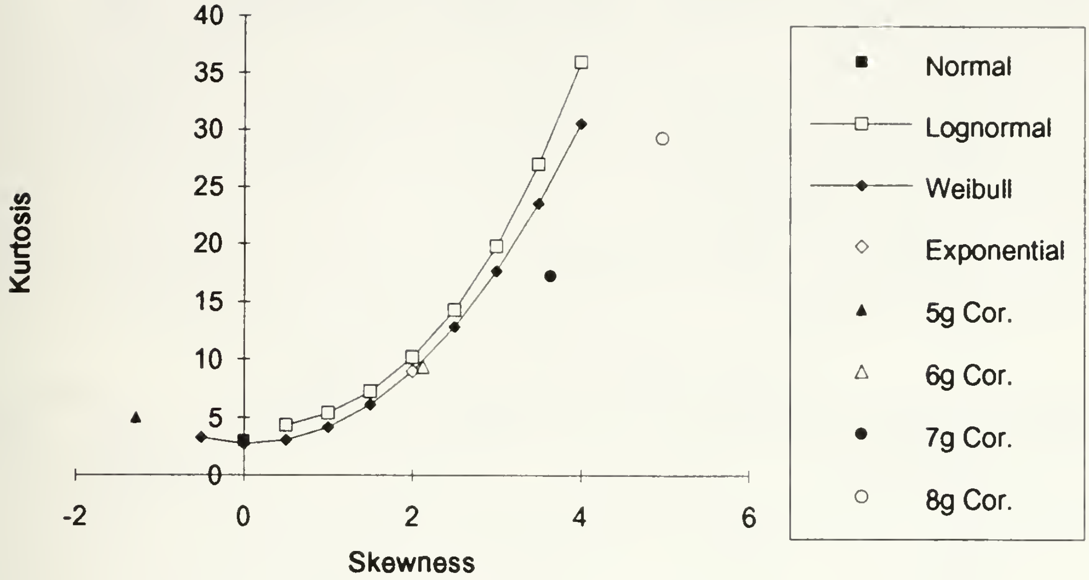


Figure 3.18: Kurt. vs. Skew. for Occurrence Data

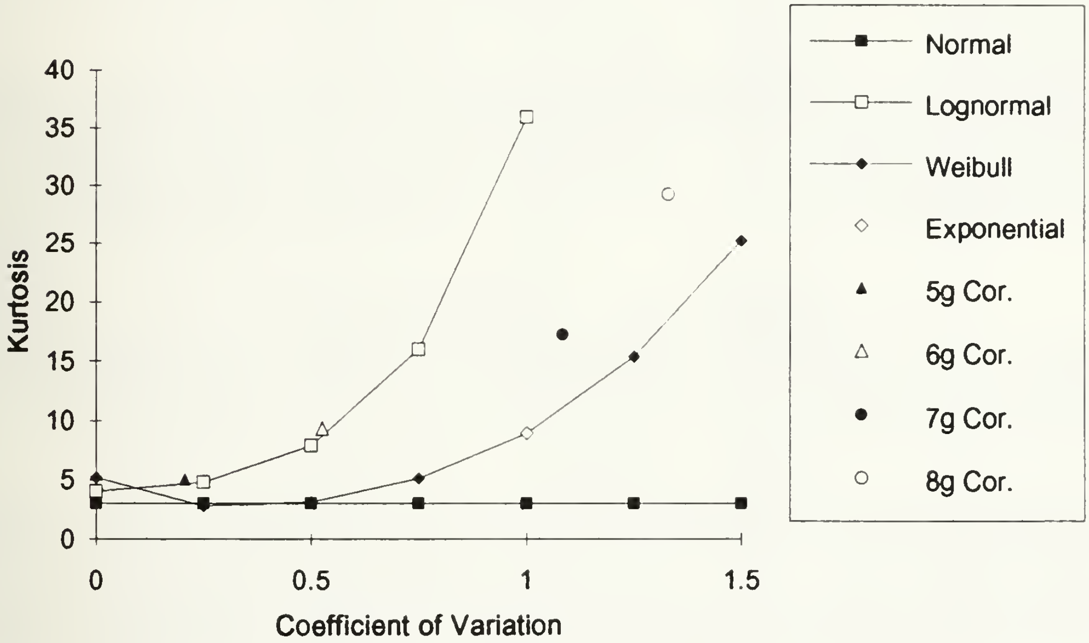


Figure 3.19: Kurt. vs. Coef. of Var. for Occurrence Data

These six figures show that the moment plots are very similar between the exceedence and occurrence data. Curve fits of the data in Figures 3.14 and 3.17 would indicate non Normal and non Exponential.

3. Goodness of Fit Results

Goodness of fit statistics were calculated for the data both in exceedence form and in occurrence form. Tables 3.45 through 3.48 are for the exceedence data. Tables 3.49 through 3.52 are for the occurrence data.

TABLE 3.45: 5G EXCEEDENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	7.7646	22.31	10.582	54.425
Significance	0.0053284	1.4318E-5	5.035E-3	9.1051E-12
Kolm-Smirn	0.13688	0.20086	0.14666	0.41431
Significance	0.42607	0.073158	0.34111	1.542E-6
Cramer-V M	0.18033	0.43461	0.19676	2.3822
Significance	>0.15	<0.1	>0.15	<0.01
Ander-Darl	1.151	2.3974	1.2571	11.361
Significance	>0.15	<0.1	>0.15	<0.01

TABLE 3.46: 6G EXCEEDENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	12.211	15.685	15.611	54.609
Significance	4.75E-4	7.48E-5	4.07E-4	8.318E-12
Kolm-Smirn	0.23798	0.19924	0.19867	0.32351
Significance	0.019235	0.077149	0.078594	0.00037489
Cramer-V M	0.58176	0.41149	0.49451	1.4046
Significance	<0.05	<0.1	<0.05	<0.01
Ander-Darl	3.361	2.1508	2.6901	6.8894
Significance	<0.025	<0.1	<0.05	<0.01

TABLE 3.47: 7G EXCEEDENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square		11.832		3.914
Significance		5.82E-4		0.047884
Kolm-Smirn	0.31918	0.34494	0.24798	0.24517
Significance	4.71E-4	1.158E-4	0.012914	0.014468
Cramer-V M	1.2139	1.537	0.74737	0.73626
Significance	<0.01	<0.01	<0.025	<0.025
Ander-Darl	6.4017	7.9169	3.8882	3.8443
Significance	<0.01	<0.01	<0.01	<0.025

TABLE 3.48: 8G EXCEEDENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				4.0212
Significance				0.044928
Kolm-Smirn	0.33601	0.42957	0.31342	0.20804
Significance	1.907E-4	5.3639E-7	6.35E-4	0.057504
Cramer-V M	1.2263	1.9683	1.0871	0.5623
Significance	<0.01	<0.01	<0.01	<0.05
Ander-Darl	6.572	10.06	6.3739	5.1045
Significance	<0.01	<0.01	<0.01	<0.01

TABLE 3.49: 5G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	5.2029	15.153	2.2213	43.361
Significance	0.074158	5.1235E-4	0.32934	3.8388E-10
Kolm-Smirn	0.12891	0.1793	0.094377	0.44802
Significance	0.50335	0.14323	0.85853	1.4221E-7
Cramer-V M	0.17213	0.43073	0.087123	2.5873
Significance	>0.15	<0.1	>0.15	<0.01
Ander-Darl	1.0794	2.5194	0.61744	12.272
Significance	>0.15	<0.05	>0.15	<0.01

TABLE 3.50: 6G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	11.499	10.298	13.126	53.55
Significance	0.0031843	0.0013314	0.0014114	1.3996E-11
Kolm-Smirn	0.20694	0.16739	0.17695	0.30846
Significance	0.059689	0.20079	0.1534	8.18E-4
Cramer-V M	0.39932	0.33598	0.36305	1.3617
Significance	<0.1	<0.15	<0.15	<0.01
Ander-Darl	2.5144	1.798	2.0962	6.7483
Significance	<0.05	<0.15	<0.1	<0.01

TABLE 3.51: 7G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square		36.482	5.3093	3.0649
Significance		1.5407E-9	0.021213	0.080001
Kolm-Smirn	0.32772	0.3547	0.28895	0.24264
Significance	2.994E-4	6.6169E-5	0.0021269	0.016009
Cramer-V M	1.0626	1.7709	0.88734	0.65679
Significance	<0.01	<0.01	<0.01	<0.025
Ander-Darl	5.7381	9.1614	4.9233	4.1693
Significance	<0.01	<0.01	<0.01	<0.01

TABLE 3.52: 8G OCCURRENCES

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square				4.0212
Significance				0.044928
Kolm-Smirn	0.33601	0.42327	0.30545	0.20804
Significance	1.91E-4	8.3351E-7	9.516E-4	0.057504
Cramer-V M	1.2263	1.9222	1.038	0.5623
Significance	<0.01	<0.01	<0.01	<0.05
Ander-Darl	6.572	9.7395	6.0296	4.8237
Significance	<0.01	<0.01	<0.01	<0.01

4. Vote Tally

Tables 3.53, 54, 55, and 56 show the distributions selected for the exceedence data. Tables 3.57, 58, 59, and 60 show the distributions selected for the occurrence data. Both have eliminated Normal and Exponential distributions based on curve fits of the data in Figures 3.14 and 3.17

TABLE 3.53: VOTES FOR 5G EXCEEDENCE

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	4	0
Vote Totals:	0	1	6	0

TABLE 3.54: VOTES FOR 6G EXCEEDENCE

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	2	1	0
Goodness of fit	0	2	2	0
Vote Totals:	0	4	3	0

TABLE 3.55: VOTES FOR 7G EXCEEDENCE

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	3	0
Vote Totals:	0	1	5	3

TABLE 3.56: VOTES FOR 8G EXCEEDENCE

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	3	0
Vote Totals:	0	1	5	0

TABLE 3.57: VOTES FOR 5G OCCURRENCES

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	4	0
Vote Totals:	0	1	6	0

TABLE 3.58: VOTES FOR 6G OCCURRENCES

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	2	1	0
Goodness of fit	0	4	0	0
Vote Totals:	0	6	1	0

TABLE 3.59: VOTES FOR 7G OCCURRENCES

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	3	0
Vote Totals:	0	1	5	0

TABLE 3.60: VOTES FOR 8G OCCURRENCES

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	0	1	2	0
Goodness of fit	0	0	3	0
Vote Totals:	0	1	5	0

The 5g, 7g, and 8g data are consistently Weibull regardless of whether the data is in exceedence or occurrence form. The 6g data are Lognormal in both cases, which is intuitively disconcerting. It was expected that all the loads would be described by the same distribution type. There was some consistency in behavior among g levels in A-6 and A-7 data. The A-6 data was Lognormal for the two lower g levels, 4g and 5g; and Weibull for the two higher g levels, 6g and 7g. The A-7 data was also Weibull for the two higher g levels, 7g and 8g; but the two lower g levels, 5g and 6g, were split between Weibull and Lognormal respectively.

B. SPECIMEN FATIGUE LIFE TESTS

The second application of the moment plot/goodness of fit vote count method was on data generated by fatigue life testing of Aluminum 7075-T6 samples.

1. Explanation of Data

The fatigue testing was originally performed to investigate the affects of mean strain on fatigue lives. Twenty samples were first tested at zero mean strain with an oscillating strain amplitude of 0.007 in/in. The fatigue lives of each sample were recorded in cycles to failure. The strain amplitude was reduced to 0.005 in/in and 20 more samples were tested and their cycles to failure recorded. This process was repeated two more times for strain amplitudes of 0.003 in/in, and 0.0025 in/in. At this point 80 lives had been recorded in terms of cycles to failure. Next, the mean strain was increased to 0.03 in/in and the testing at the four strain amplitudes described above was repeated producing 80 more fatigue lives. Two more mean strains were tested in this manner: 0.063 in/in and 0.100 in/in. A total of 16 combinations of mean strain and strain amplitude were tested, with 20 samples for each combination producing 320 fatigue lives. [Ref. 11]

2. Moment Plots

The AGSS analysis was made on each of the 16 sets of lives, producing values for the coefficient of variation, skewness, kurtosis, and goodness of fit statistics. All 16 tests are shown together on Figures 3.20, 3.21, and 3.22. Each point represents 20 samples tested at a given combination of mean strain and strain amplitude. The trend in Figure 3.20 is too close to differentiate Weibull from Normal, but the negative values of skewness in Figure 3.21 and the trend in Figure 3.22 eliminate Lognormal as a candidate.

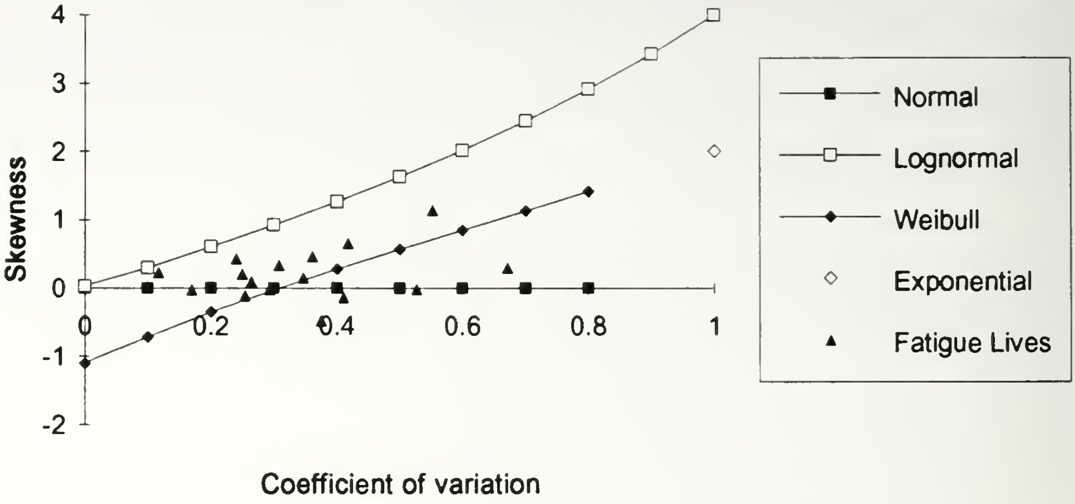


Figure 3.20: Specimen Skewness vs. Coefficient of Variation

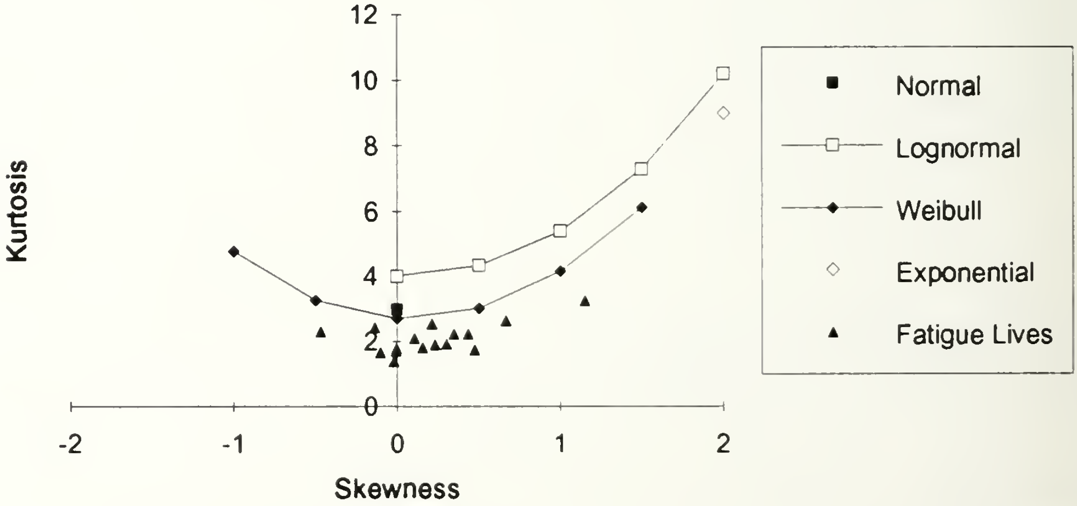


Figure 3.21: Specimen Kurtosis vs. Skewness

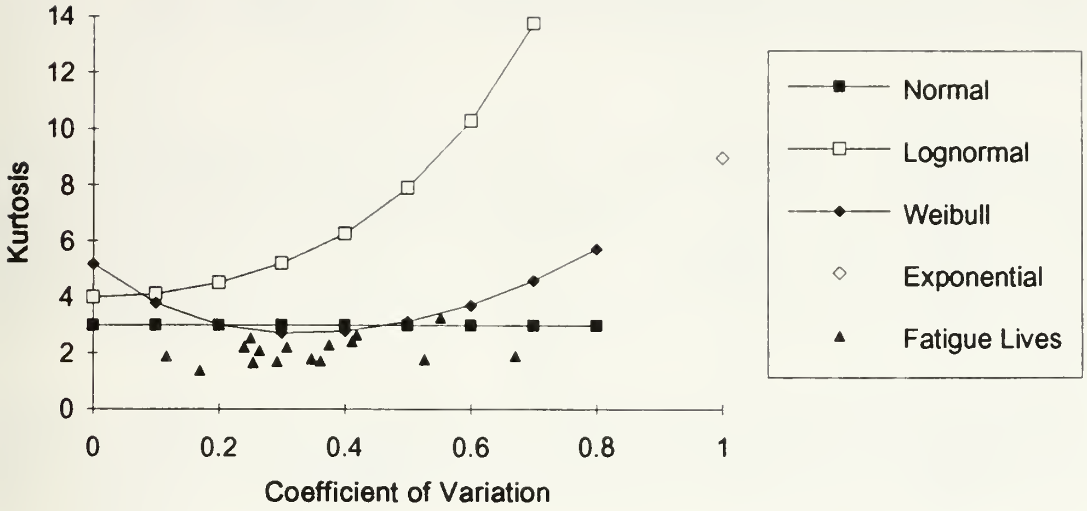


Figure 3.22: Specimen Kurtosis vs. Coefficient of Variation

Moment plots were also made for two different sub poolings of this data to see if mean strain or strain amplitude had an affect on the distribution of lives. Four pools, each consisting of tests at the same mean strain and varying strain amplitudes were plotted first. Then four pools, each consisting of tests at the same strain amplitude and varying mean strains were plotted. No noticeable trends or groupings resulted from sub pooling the data; therefore, the plots resulting from a single pool of all the data were used in the vote counting.

3. Goodness of Fit Results

Tables 3.60, through 3.75 contain the goodness of fit statistics. Each table represents a test of 20 samples.

Table names correspond to variable names assigned in the fatigue tests.

TABLE 3.60: MEAN STRAIN = 0.0, STRAIN AMPLITUDE = .007

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	3.5466		3.5887	20.827
Significance	0.059668		0.058173	0.000030036
Kolm-Smirn	0.14705	0.18669	0.15221	0.34879
Significance	0.78012	0.48855	0.74309	0.015404
Cramer-V M	0.053836	0.17885	0.061525	0.7073
Significance	>0.15	>0.15	>0.15	<0.025
Ander-Darl	0.38297	1.1713	0.49698	3.5822
Significance	>0.15	>0.15	>0.15	<0.025

TABLE 3.61: MEAN STRAIN = 0.0, STRAIN AMPLITUDE = .005

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	4.6521	6.0559	4.3013	15.034
Significance	0.097689	0.048413	0.11641	0.00010556
Kolm-Smirn	0.13975	0.18006	0.13348	0.43865
Significance	0.82959	0.53561	0.86829	0.0009087
Cramer-V M	0.074173	0.091191	0.065225	0.93137
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.45714	0.48564	0.41985	4.5702
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.62: MEAN STRAIN = 0.0, STRAIN AMPLITUDE = .003

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	3.8739	4.6416	3.3067	4.0283
Significance	0.14414	0.098203	0.19142	0.044739
Kolm-Smirn	0.19126	0.16504	0.18408	0.43859
Significance	0.45727	0.64729	0.50683	0.0009107
Cramer-V M	0.14625	0.10164	0.12839	0.80802
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.89393	0.68521	0.78966	4.0295
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.63: MEAN STRAIN = 0.0, STRAIN AMPLITUDE = .0025

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	7.2084		6.93	
Significance	0.0072559		0.0084761	
Kolm-Smirn	0.18522	0.1823	0.18653	0.54473
Significance	0.4988	0.5195	0.48968	0.000014006
Cramer-V M	0.13963	0.15234	0.12818	1.3708
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.95282	1.008	0.90021	6.4406
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.64: MEAN STRAIN = 0.03, STRAIN AMPLITUDE = .007

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	0.31305	0.46384	0.18183	18.029
Significance	0.57582	0.49584	0.66981	0.000021744
Kolm-Smirn	0.083576	0.090969	0.074902	0.42999
Significance	0.99902	0.99643	0.99987	0.0012279
Cramer-V M	0.020875	0.034031	0.017834	1.0366
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.15737	0.23051	0.14231	5.0263
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.65: MEAN STRAIN = 0.03, STRAIN AMPLITUDE = .005

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	0.73182	1.6243	0.6278	19.722
Significance	0.69356	0.4439	0.73059	0.0000089624
Kolm-Smirn	0.097971	0.10881	0.11788	0.43353
Significance	0.99074	0.97186	0.94385	0.0010867
Cramer-V M	0.024398	0.042895	0.032707	1.1066
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.21596	0.27841	0.26395	5.3126
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.66: MEAN STRAIN = 0.03, STRAIN AMPLITUDE = .003

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.6331	1.568	1.3487	
Significance	0.44194	0.45656	0.50948	
Kolm-Smirn	0.095384	0.097991	0.10933	0.56545
Significance	0.99332	0.99072	0.97058	0.0000055803
Cramer-V M	0.035669	0.030705	0.054553	1.5594
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.30757	0.27732	0.40755	7.2249
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.67: MEAN STRAIN = 0.03, STRAIN AMPLITUDE = .0025

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.2513	1.8817	2.3697	1.1522
Significance	0.32445	0.17014	0.3058	0.56208
Kolm-Smirn	0.12538	0.15957	0.15015	0.1687
Significance	0.91166	0.68846	0.75805	0.61969
Cramer-V M	0.063161	0.12833	0.070748	0.14263
Significance	>0.15	>0.15	>0.15	>0.15
Ander-Darl	0.4721	0.82801	0.5317	0.88894
Significance	>0.15	>0.15	>0.15	>0.15

TABLE 3.68: MEAN STRAIN = 0.063, STRAIN AMPLITUDE = .007

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.1943	1.2046	1.0757	12.375
Significance	0.55037	0.54754	0.58401	0.00043518
Kolm-Smirn	0.10062	0.12585	0.098234	0.41914
Significance	0.98742	0.90937	0.99045	0.0017748
Cramer-V M	0.04696	0.056594	0.045363	0.93811
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.33797	0.39549	0.32437	4.6227
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.69: MEAN STRAIN = 0.063, STRAIN AMPLITUDE = .005

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.1926	1.5325	1.0206	11.518
Significance	0.27482	0.21574	0.31239	0.0031554
Kolm-Smirn	0.15906	0.10905	0.13886	0.34888
Significance	0.69232	0.97128	0.83532	0.015365
Cramer-V M	0.07052	0.030827	0.052772	0.66747
Significance	>0.15	>0.15	>0.15	<0.025
Ander-Darl	0.47108	0.21421	0.34744	3.3756
Significance	>0.15	>0.15	>0.15	<0.025

TABLE 3.70: MEAN STRAIN = 0.063, STRAIN AMPLITUDE = .003

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.2688	2.3519	2.2559	2.7812
Significance	0.32162	0.30854	0.32371	0.24892
Kolm-Smirn	0.1289	0.16171	0.11951	0.24021
Significance	0.89383	0.67234	0.93755	0.19872
Cramer-V M	0.043051	0.11781	0.048927	0.33541
Significance	>0.15	>0.15	>0.15	<0.15
Ander-Darl	0.37518	0.74441	0.41593	1.8466
Significance	>0.15	>0.15	>0.15	<0.15

TABLE 3.71: MEAN STRAIN = 0.063, STRAIN AMPLITUDE = .0025

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	0.41296	0.79929	0.39988	6.4534
Significance	0.52047	0.37131	0.52715	0.039684
Kolm-Smirn	0.069334	0.14717	0.074689	0.3133
Significance	0.99998	0.77927	0.99988	0.039431
Cramer-V M	0.01504	0.10795	0.01976	0.61633
Significance	>0.15	>0.15	>0.15	<0.05
Ander-Darl	0.11644	0.74452	0.17638	3.1074
Significance	>0.15	>0.15	>0.15	<0.025

TABLE 3.72: MEAN STRAIN = 0.100, STRAIN AMPLITUDE = .007

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.1205	0.075459	1.0053	12.223
Significance	0.57106	0.78355	0.60493	0.0022174
Kolm-Smirn	0.10926	0.12072	0.10602	0.37787
Significance	0.97075	0.93264	0.97814	0.0066163
Cramer-V M	0.052007	0.03839	0.047708	0.78804
Significance	>0.15	>0.15	>0.15	<0.025
Ander-Darl	0.36605	0.3184	0.3333	3.9635
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.73: MEAN STRAIN = 0.100, STRAIN AMPLITUDE = .005

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	1.2459	0.056647	1.3579	35.226
Significance	0.53636	0.81188	0.50716	2.9362E-9
Kolm-Smirn	0.11135	0.10215	0.1156	0.4644
Significance	0.96524	0.98514	0.95204	0.00035855
Cramer-V M	0.053202	0.021513	0.06938	1.1512
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.40301	0.20646	0.48192	5.4859
Significance	>0.15	>0.15	>0.15	<0.01

TABLE 3.74: MEAN STRAIN = 0.100, STRAIN AMPLITUDE = .003

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.2149	3.0192	1.9065	2.4968
Significance	0.33041	0.082286	0.38549	0.11408
Kolm-Smirn	0.19364	0.18398	0.17357	0.34374
Significance	0.44134	0.50758	0.58327	0.017719
Cramer-V M	0.17651	0.083651	0.11783	0.49407
Significance	>0.15	>0.15	>0.15	<0.1
Ander-Darl	1.1856	0.54737	0.80575	2.5956
Significance	>0.15	>0.15	>0.15	<0.05

TABLE 3.75: MEAN STRAIN = 0.100, STRAIN AMPLITUDE = .0025

Test \ Fit	Normal	Lognormal	Weibull	Exponential
Chi-Square	2.1391	3.146	2.2706	11.338
Significance	0.34316	0.20743	0.32133	0.00075919
Kolm-Smirn	0.13182	0.13776	0.12351	0.44985
Significance	0.87784	0.84229	0.92045	0.00061036
Cramer-V M	0.059801	0.070682	0.060708	1.0615
Significance	>0.15	>0.15	>0.15	<0.01
Ander-Darl	0.46153	0.50914	0.46891	5.152
Significance	>0.15	>0.15	>0.15	<0.01

4. Vote Tally

Table 3.76 contains the vote count results for the fatigue life data. Since all the parameters were pooled, there is just one table. Data trends from the moment plots have eliminated Lognormal and Exponential; therefore, these distributions were eliminated in tallying the goodness of fit results.

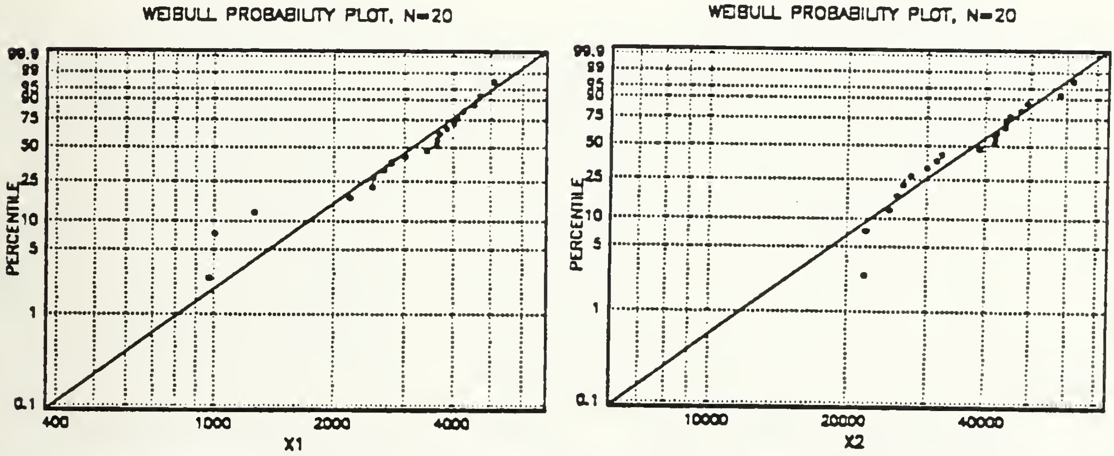
TABLE 3.76: FATIGUE LIFE VOTES

TEST\DISTRIBUTION	Normal	Lognormal	Weibull	Exponential
Moment Plots	14	0	35	0
Goodness of fit	26	0	37	0
Vote Totals:	40	0	72	0

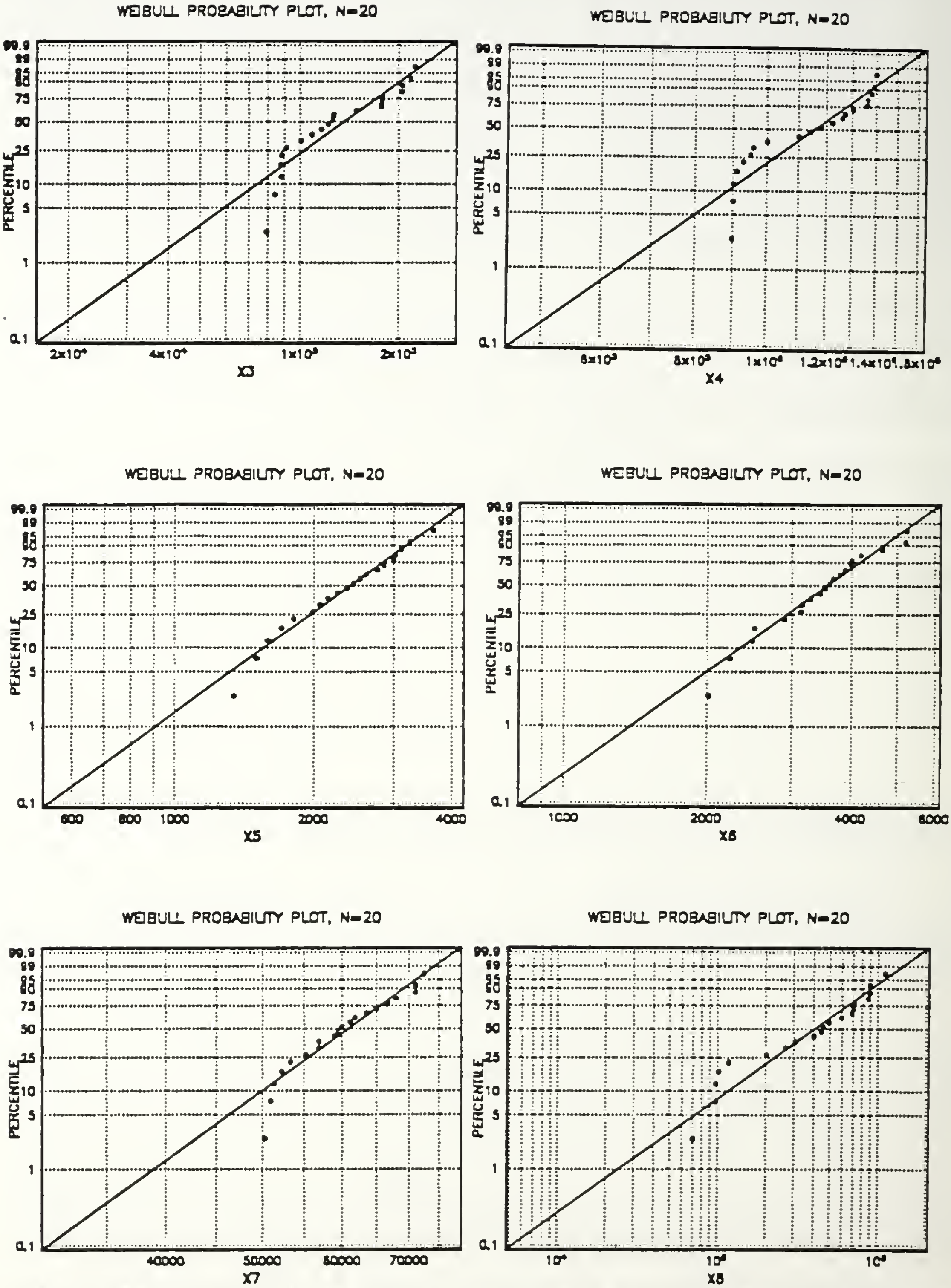
Based on the results of the random number testing of this method, a nearly 2:1 margin of Weibull votes over Normal votes is a strong indication that the samples come from a population with a Weibull distribution.

5. Weibull Zero-Shift

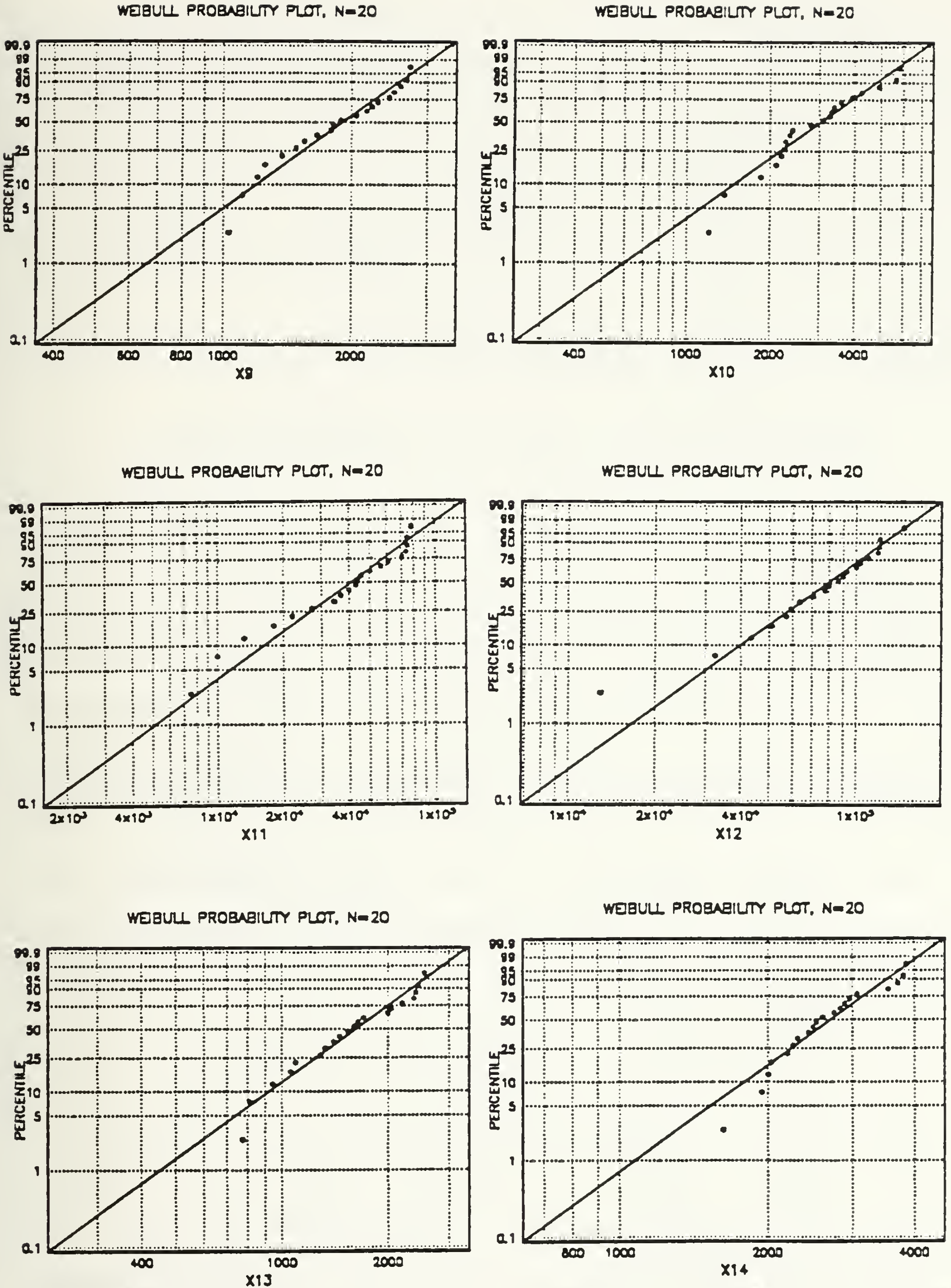
Probability plots from the Weibull fit of each of the 16 sets of 20 samples are shown in Figures 3.77a through 3.77p. The plots are in the same order as the preceeding tables, so the mean strain and strain amplitude can be referenced. When AGSS fits a sample to a Weibull distribution, it calculates the parameters of the Weibull distribution as well. To develop a model for the lives, these parameters are used. However, the fit for some of these samples can be improved by shifting the x-axis. Samples that have points dropping below the line at the lower tail indicate that a "zero shift" would improve their fit.



Figures 3.77a, b: Weibull Probability Plots



Figures 3.77c, d, e, f, g, h: Weibull Probability Plots



Figures 3.77i, j, k, l, m, n: Weibull Probability Plots

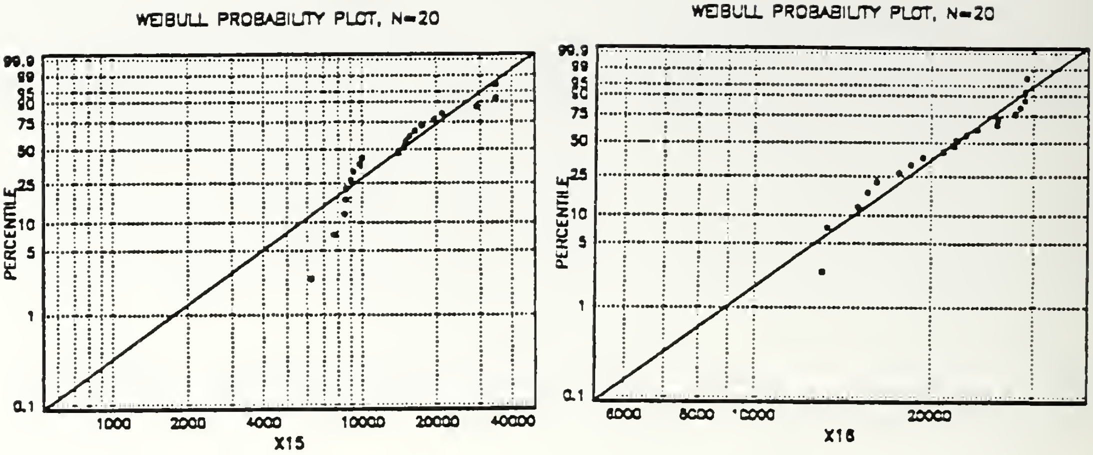


Figure 3.77o, p: Weibull Probability Plots

The plot of Figure 3.77g will be used as an example of how the zero-shift is accomplished. The curve of the left tail is extrapolated down to the x-axis as is done on Figure 3.77g. The x-intercept is the amount that should be subtracted from the x-values, effectively shifting the zero to the intercept. Figures 3.78a, b, c, d, e, and f, show how the Weibull probability plots are affected by zero-shifts of -40,000, -42,000, -44,000, -46,000, -48,000, and -50,000 respectively.

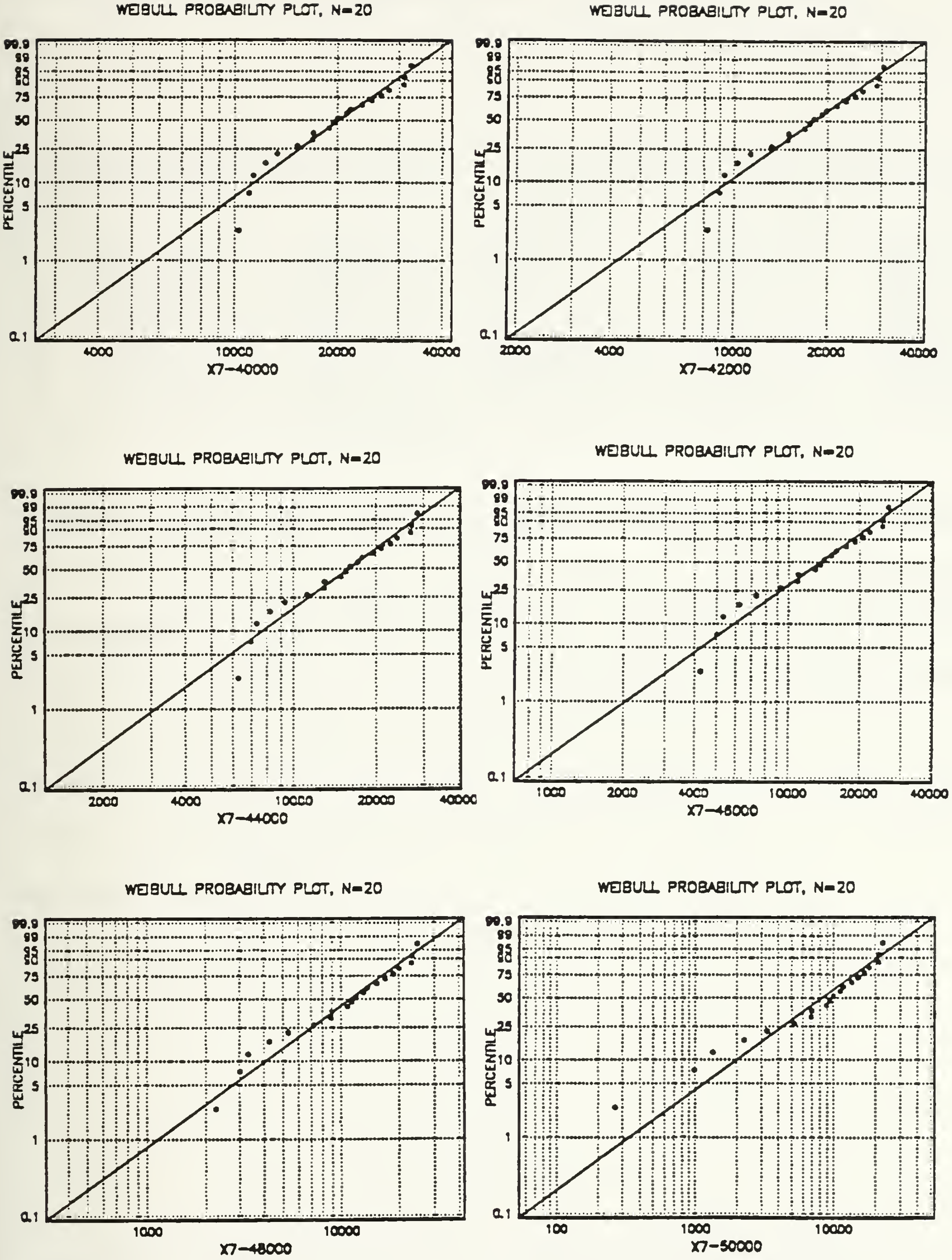


Figure 3.78a, b, c, d, e, f: Weibull Probability Plots

Table 3.77 contains the goodness of fit statistics for the various zero-shifts.

TABLE 3.77: GOODNESS OF FIT VALUES FOR ZERO-SHIFTS

SHIFT\TEST	**Chi-Square**	**Kolm-Smirn**	**Cramer-V.M.**	**Ander-Darl**
None	1.3487	0.10933	0.054553	0.40755
-40,000	1.2295	**0.090134**	0.031382	0.27399
-42,000	1.2805	0.091916	0.029391	0.26238
-44,000	1.2616	0.09371	0.027817	0.25322
-46,000	1.7745	0.09503	**0.027611**	**0.25214**
-48,000	**0.52033**	0.093812	0.031944	0.27793
-50,000	1.3023	0.12578	0.062855	0.45714

From the table it appears that the shift that would produce the lowest values of the goodness of fit statistics is approximately -46,000. Figure 3.79 is a graphical representation of Table 3.77.

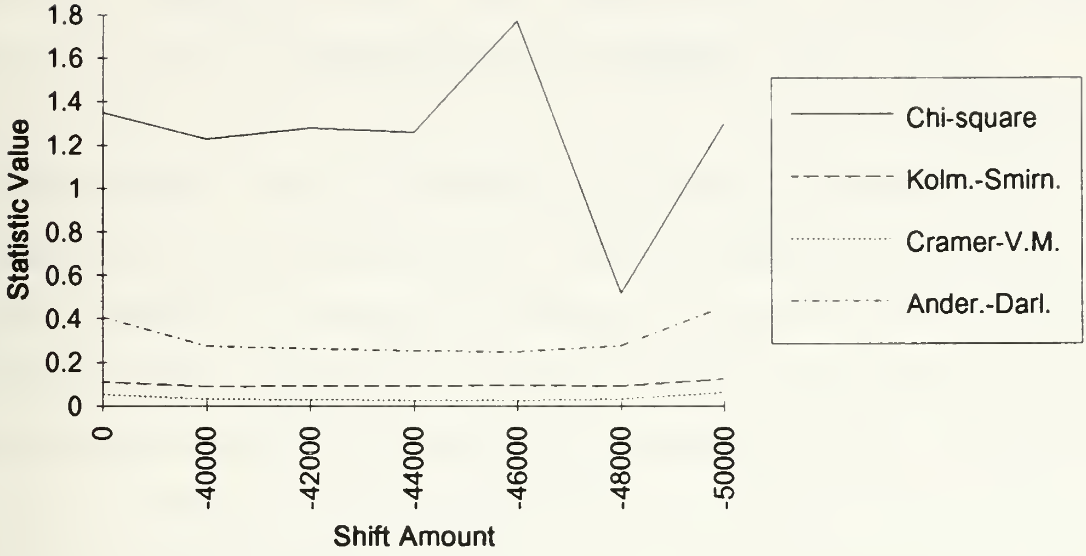


Figure 3.79: Goodness of Fit Statistics vs. Shift Amount

AGSS calculates shape and scale parameters of 2.2377 and 16206 respectively when it fits the X7 data set to a Weibull distribution that is shifted to the left 46,000. To model the population of lives this data set is drawn from, these parameters would be used to generate lives from a Weibull distribution.

IV. SUMMARY AND CONCLUSIONS

A. SUMMARY

The moment plotting method of distribution characterization was developed by first deriving expressions for the coefficient of variation, measure of skewness, and measure of kurtosis, for the Normal, Lognormal, Weibull, and Exponential distributions. The derived expressions were plotted three ways as templates: (1) skewness versus coefficient of variation; (2) kurtosis versus skewness; and (3) kurtosis versus coefficient of variation. This produced three templates of moment curves. Before applying the method to fatigue data, the moment method was tested on random samples drawn from known distributions to determine whether or not it would correctly characterize the samples as being from the distribution they were drawn from.

The first test used sample sizes of 1,000 and plotted sample moments on the templates. Points for the sample moments did not fall directly on any of the template curves, but trends were evident what were helpful in selecting the correct distribution. Template (1) exhibited horizontal trends in the data distribution for normal samples, while Lognormal and Weibull exhibited data trends with a significant positive slope. Templates (2) and (3) were useful in distinguishing between Lognormal and Weibull distributions. Template (2) was distinguishing on the negative skewness axis and template (3) possessed the largest spatial distance between Lognormal and Weibull.

A vote counting method was also devised, where each plotted point voted for the distribution curve the point lay closest to on the template. Using this technique, the moment method correctly identified most of the sample distributions correctly; however, sometimes it did not and sometimes it was by a very narrow margin. When the sample size was reduced to 100 and then to 20, the method grew more and more unreliable. A weakness of the moment plotting method was the fact that the moment functions forming the template were very near one another, and when the sample moments were plotted, they often spilled into the domain of more than one moment.

The moment plots were useful, however, in eliminating distributions from consideration by observing the trends of the data and how they compared to the random samples initially tested. In most of the applications there was at least one distribution, and often times there were two, that could be ruled out as having trends unlike the data. This was the case in every one of the applications considered here. For instance, in the case of the helicopter loads data the Lognormal and Exponential distributions were dropped, reducing the task to choosing between the remaining two.

To improve the selection accuracy, goodness of fit statistics were coupled with the moment plotting method. Each sample was fit to a trial distribution using AGSS. For each fit, AGSS calculated the Chi-square, Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness of fit statistics. The goodness of fit measures did not always agree among themselves, since they are measures of quite different things, and they were often times at odds with results from the locations on the moment plots, so the vote counting method was incorporated to make the final determination of the best distribution from the group remaining after the moment plotting method was used to make the first elimination(s).

Applications were successfully made of the method to characterize the probability distribution functions of helicopter loads data, two different sets of tactical aircraft maneuver data, and a large set of experimental measurements of fatigue lives using uniaxial specimens.

B. CONCLUSIONS

Neither goodness of fit statistics nor the moment plotting method by themselves could consistently pick the correct distribution for a sample drawn from a known population. On the other hand when both were used together, employing the method outlined in the thesis, the success rate was 100%.

The method was applied to a broad range of fatigue data and found to be applicable.

Applying the method to the several varied fatigue applications, there was considerable confidence in the distribution selected because of the preponderance of votes for that distribution by all the measures used in the final steps after the initial distributions were weeded out using the moment plotting method.

This method, which employs both the trends that can be observed from the moment plotting method and the voting approach of the moments and the goodness of fit measures, seems to be considerably better than any of the measures used alone in a classical manner.

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